The Effects of Teaching a Mathematics Problem Solving Strategy on The Problem Solving Ability of Grade Nine Students

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Abstract

Forty grade 9 students were selected from a small rural board in southern Ontario. The students were in two classes and were treated as two groups. The treatment group received instruction in the Logical Numerical Problem Solving Strategy every day for 37 minutes over a 6 week period. The control group received instruction in problem solving without this strategy over the same time period. Then the control group received the treatment and the treatment group received the instruction without the strategy.

Quite a large variance was found in the problem solving ability of students in grade 9. It was also found that the growth of the problem solving ability achievement of students could be measured using growth strands based upon the results of the pilot study. The analysis of the results of the study using t-tests and a MANOVA demonstrated that the teaching of the strategy did not significantly (at $p \leq 0.05$) increase the problem solving achievement of the students. However, there was an encouraging trend seen in the data.

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CHAPTER ONE: THE PROBLEM

Introduction

Mathematics education has evolved from basic skill development with supplemental problems, to problem solving becoming the central focus (National Council of Teachers of Mathematics, 1989; Ontario Association for Mathematics Education & Ontario Mathematics Coordinators Association, 1993). However, research on the teaching and assessment of mathematics problem solving has not kept pace with the emphasis in mathematics education. Instead, research has concentrated on understanding the nature of mathematics problem solving (Charles & Silver, 1988; Schoenfeld, 1985; Silver, 1988). More research in teaching and assessment is needed to promote growth in the problem solving ability of students.

This is a study of mathematics problem solving: the evaluation of mathematics problem solving, the development of growth strands and the effects of teaching a mathematics problem solving strategy on problem solving ability.

Background

Problem solving has become the focus of mathematics in the nineties. The National Council of Supervisors of Mathematics (NCSM) (1988) said that "learning to solve problems is the principal reason for studying mathematics" (p. 2). In their document titled, Curriculum and Evaluation Standards for School Mathematics (1989), the National Council of Teachers of Mathematics (NCTM) reiterated their earlier statement that "Problem solving must be the primary goal of school mathematics" (p. 6).

In 1993, the Ontario Association for Mathematics Education (OAME) and the Ontario Mathematics Coordinators Association (OMCA) released the document, Focus on Renewal of Mathematics Education: Guiding Principles for the Early, Formative and Transition Years, which emphasizes the centrality of problem solving in the mathematics curriculum. Later in 1995 the Ontario Ministry of Education and Training (OMET) published the document titled, The Common Curriculum: Polices and outcomes, grades 1-9 (1995a). It states that "learning is an active process...curriculum should encourage this kind of constant inquiry" (p. 17) and "the processes of inquiry and problem solving are basic to the study of mathematics, science, and technology" (p. 79). These views clearly reflect the recognition of the value of problem solving and its central role in mathematics education.

In the spring of 1990 a study of mathematics education in grades 6, 8, 10 and 12 was completed by the Ontario Ministry of Education (OME, 1990a, 1990b, 1990c, and 1990d). This report attempted to answer two main questions: "How well are students in these grades achieving the goals of the mathematics curriculum?" and "How are students in these grades taught mathematics?" (OME, 1990a, 1990b, 1990c, 1990d). The results indicated a weakness in solving process problems (applying problem solving strategies to unfamiliar problems). In all four grades fewer than 50% (40% in grade 6; 36% in grade 8; 33% in grade 10; 47% in grade 12) of the responses to these questions were correct. This was low in relation to the other areas of mathematics measured. Since the Ministry of Education in Ontario (1985) has stated that "developing problem solving ability is a major goal of mathematics education" this report clearly indicates a need to concentrate research in this area. This study is a move in the recommended research direction.

In 1995, the Ontario Ministry of Education and Training released the document, The Common Curriculum: Provincial standards: Mathematics, grades 3, 6, 9 (1995b), which stated that "problem solving should be the central focus of the mathematics program" (p. 11). As a result problem solving will become a focus in mathematics in these grades.

However, there is little information about how to teach and evaluate mathematics problem solving. This information is critical to realizing positive gains in mathematics problem solving ability in students.

In the rapidly changing global economy and changing work force problem solving is becoming even more important for students. The emphasis on manipulatives in mathematics, for concept development, uses problem solving as the means to develop these concepts. With the focus on problem solving (OAME & OMCA, 1993; OMET, 1995a) and the inclusion of problem solving in the Ministry's image of the learner, there will continue to be a major emphasis on problem solving in mathematics education.

Purpose

The purpose of this study is threefold: to determine the range in performance of students in problem solving, to evaluate growth in problem solving ability of students using growth strands for the logical numerical problem solving strategy, and to determine the effect of teaching a mathematics problem solving strategy on students' ability to solve problems in mathematics.

Problem Statements

This paper attempts to answer the following questions about mathematics problem solving:

1. What is the range of problem solving ability levels for students in the intermediate division?

Through the administration of an evaluation instrument this question investigates the range of mathematics problem solving ability in students in the grade 9 classroom.

2. What are the effects of being taught a problem solving strategy upon students' abilities to solve problems?

This question investigates the effects on the problem solving ability of grade 9 students after they have been taught the logical numerical problem solving strategy.

3. Can growth in problem solving be evaluated at each stage of the problem solving process utilizing growth strands for the logical numerical problem solving strategy?

This question investigates the use of series of observable behaviours of grade 9 students during the problem solving process to evaluate growth in mathematics problem solving ability.

Rationale

Problem solving has become a focus for mathematics education in Ontario (OAME & OMCA, 1993; OMET, 1995a, 1995b) and the United States (NCSM, 1988; NCTM, 1989). However, the main emphasis in the classroom is still on skill development with some problem solving. This is slowly beginning to change but there is a need to develop teaching strategies and assessment techniques in the classroom. These would assist teachers in establishing problem solving as the central focus and would facilitate the growth of problem solving ability of students.

There are few research studies designed to investigate the effect of teaching a problem solving strategy on students' problem solving ability (Charles & Lester, 1985; Schoenfeld, 1982). Research in this field is needed to determine the effectiveness of one strategy which might be used by educators.

The metacognitive aspect of problem solving has been recognized by many researchers of problem solving (Brown, 1978; Charles & Lester, 1985; Charles & Silver, 1988; Fortunato, Hecht, Tittle, & Alverez, 1991; Lester, 1988; Schoenfeld, 1985). Metacognition refers to being cognizant of one's own cognitive processes and products. It is an awareness of our thinking processes as we perform a task and the control of these processes.

Problem solving requires a metacognitive strategy to be effective. Schoenfeld (1982) calls it a "managerial strategy". As a result teachers need to emphasize the metacognitive aspect of problem solving to have students improve their problem solving ability. It is this thinking process which the student must go through when solving the problems that is so difficult to teach and evaluate.

This study utilizes growth strands to evaluate mathematics problem solving and investigates the teaching and assessment of mathematics problem solving in an effort to provide information for educators to improve students' problem solving ability.

Importance of the Study

Problem solving is a major goal of mathematics education (OAME & OMCA, 1993; OME, 1985; OMET, 1995b). As a result the focus of the mathematics curriculum for all grades in Ontario will eventually be problem solving. The questions related to this are "How can we teach our students?" and "How can we evaluate their progress?" The development of effective teaching strategies will be realized only by having effective evaluation.

This study will provide teachers, researchers, and administrators with information to assist them in facilitating the development of effective teaching strategies and observing cognitive growth in students' problem solving ability. With the emphasis on mathematics problem solving in Ontario classrooms, the need to develop effective strategies and growth strands will continue to be crucial for mathematics education.

Defining Problem Solving

Problem solving in mathematics is an area which has been receiving increased attention in recent years (Charles, Lester & O'Daffer, 1987; Charles & Silver, 1988; Krulick & Rudnick, 1989; Lester, 1988; Schoenfeld, 1985; Silver, 1988). However, the lack of a universally agreed upon definition of "problem" has resulted in different definitions and interpretations of problem solving in the classroom.

In mathematics textbooks the traditional definition of problem is found within the "word problems" at the end of each chapter (Alexander, Folk, Worth & Cowan, 1990; Dottori, Knell, Lessard, McPhail & Collins, 1987). Students solve these "problems" by applying algorithms or concepts learned in the chapter. Some educators (Kantowski, 1980; Krulick & Rudnick, 1989) would call these exercises and not word problems at all.

In Krulick and Rudnick's book (1989) a distinction is made between question, exercise, and problem:

1. A question is a situation which can be completed by recalling information from memory and presenting it.

2. An exercise is a situation in which a previously learned skill or algorithm is applied to obtain the solution.

3. A problem is a situation that requires analysis and synthesis of prior knowledge to resolve it.

There is no need for the student to apply any higher order thinking to complete exercises correctly since they are applying a known algorithm or skill. When students are confronted with a "problem" they do not have a predesigned algorithm to "fit" the situation, or they do not know beforehand which algorithms are appropriate, and have a difficult time generating a solution.

In the NCTM 1980 yearbook Branca describes the most common interpretations of problem solving as a goal in itself, as a process, and as a basic skill. The NCTM (1989), OAME and OMCA (1993) and the NCSM (1988) view problem solving as a process. Results from research describe problem solving as a process that develops over a long period of time (Kantowski, 1980, 1981).

There are school (textbook) problems, real life problems, and unique (nonroutine) problems which all require consideration when developing a definition of "problem". Before a definition could be constructed, a decision was made about the range of problems it should pertain to and what its use would be. The definition for "problem" which will be a basis for this study is a situation for which the individual sees no apparent path to a solution at the onset. For the purposes of this study the definition of problem solving is given below:

> Problem solving is the process of coordinating and applying prior knowledge, experience, skills, understanding, and intuition in an effort to determine a solution for a situation.

Definition of Terms

Basic Thinking Skills are the cognitive skills at the core of all learning: observation, classification, seriation, and correspondence.

Evaluation is the making of judgements based on comparison of established measurements against specific criteria.

Intermediate Division includes students enrolled from grades 7 to 10, generally, the 12 to 15 year old student.

Metacognition is being cognizant of one's own cognitive processes and products.

Growth Strand is a series of levels of observable behaviours describing changes in the complexity of a skill. It is an elaboration of the learnings required for a specific task.

Mathematics Problem Solving is the process of coordinating and applying prior knowledge, experience, skills, understanding, and intuition in an effort to determine a solution for a problem.

Logical Numerical Problem Solving Strategy is an organized sequence of procedures for solving problems in mathematics. It is a model for directing the process of problem solving, designed to represent the problem solving heuristic.

Heuristic is a mental representation which provides direction and develops understanding and performance in a complex process (in this case, solving problems). The use of heuristics increases the probability that a solution for a problem will be discovered.

Strategy is the practical application of the heuristic in solving problems.

Scope and Delimitations of Study

This study investigated the possible range of problem solving ability in students, the effects of being taught a problem solving strategy upon student abilities to solve problems, and the observation of cognitive growth in problem solving. The study looks specifically at a small group of grade 9 students in a small rural board in southern Ontario.

The study does not profess to find the most effective method of teaching problem solving nor does it investigate the most effective evaluation technique. These are questions which should be investigated in further studies. This study investigates the effects of teaching one particular strategy (the logical numerical problem solving strategy) and an evaluation technique employing growth strands. It provides information that would benefit educators and researchers in the field of mathematics problem solving.

Summary

The description of the background and rationale for the problem demonstrated the centrality of problem solving in mathematics education and the need for research in the areas of assessment and teaching of mathematics problem solving. This study was undertaken because of the general need for further research in these areas. The purpose of this study is the teaching of the logical numerical problem solving strategy and the evaluation of mathematics problem solving ability utilizing growth strands. From this purpose three problem statements were generated to provide a focus for the study:

1. What is the range of problem solving ability for students?

2. What are the effects of being taught the logical numerical problem solving strategy upon students' ability to solve problems?

3. Can cognitive growth in problem solving be evaluated at each stage of the problem solving process utilizing growth strands developed for the logical numerical problem solving strategy?

Overview of the Study

Chapter One has outlined the intent of the study. Chapter Two reviews the related research and discusses the nature of mathematics problem solving, the generic problem solving strategy, and problem solving in mathematics. Chapter Three includes the hypotheses and describes the experimental design, sample, and the methodology and procedures used in the study. Chapter Four provides a description of the results from the study. Chapter Five describes the nature of the findings, conclusions, and suggestions for further research. References, a bibliography, and appendices follow these chapters.

CHAPTER TWO: REVIEW OF THE RELATED LITERATURE

Introduction

There is a significant number of research studies designed to investigate mathematics problem solving (Charles & Silver, 1988; Lester, 1988; Schoenfeld, 1985; Silver, 1988). Recently a considerable amount of research has focussed on understanding problem solving and determining how to develop this in students (Charles & Lester, 1985; Kantowski, 1980, 1981; Krulick & Rudnick, 1989; Lester, 1988; Schoenfeld, 1985; Silver, 1988). Both the November 1977 issue of the Arithmetic Teacher and The National Council of Teachers of Mathematics 1980 Yearbook contain articles pertaining only to mathematics problem solving. However, even though research has focussed on problem solving, there are few studies designed to measure problem solving ability (Charles, Lester & O'Daffer, 1987; Malone, Douglas, Kissane & Mortlock, 1980; Schoenfeld, 1985, 1988) and the teaching of a problem solving strategy.

This chapter will first present a historical overview of the research on the teaching and measurement of mathematics problem solving. It will then examine the nature of problem solving in mathematics, and discuss the generic problem solving strategy and problem solving in mathematics.

Historical Overview

Over the past two decades a considerable amount of progress has been made in understanding the nature of mathematics problem solving (Schoenfeld, 1985; Silver, 1988). However, much less is known about the assessment and teaching of mathematics problem solving.

A few articles have investigated the assessment of mathematics problem solving performance (Charles et al., 1987; Malone et al., 1980; Schoenfeld, 1985; Silver & Kilpatrick, 1988) and the teaching of mathematics problem solving (Campione, Brown & Connell, 1988; Charles & Lester, 1985; Charles & Silver, 1988; Davis & McKillip, 1980; Jacobson, Lester & Stengel, 1980; Kantowski, 1980; LeBlanc, Proudfit & Putt, 1980; Marshall, 1988; Pólya, 1957; Schoenfeld, 1985).

The teaching of problem solving in mathematics should focus on a problem solving model. This would provide the students with a plan to solve problems. The articles above have used Pólya's model as the basis for their research. The generic problem solving strategy (Popp, Robinson & Robinson, 1975) expands upon the model by Pólya and is currently used in many Ontario classrooms. However, research on the teaching of this model is not provided in the literature; but the investigation of the methodology following Pólya's model could be used as a basis for the teaching of the generic problem solving strategy.

Improvement of students' performance in problem solving requires much more than simply giving them many problems to solve. Problem solving is a very complex process that requires the teacher to be familiar with the model and the nature of problem solving. Teachers need to have a proper definition of a problem and three basic assumptions about problem solving before they can begin to teach problem solving (Kantowski, 1980):

1. Problem solving is for everyone.

2. Growth in problem solving ability is the result of carefully planned instruction.

3. Problem solving develops slowly over a long period of time.

Kantowski (1980) believes that these three assumptions are important for a teacher of problem solving. The second assumption would result in carefully planned lessons instead of simply assigning many problems to complete in class. Schoenfeld (1985) suggests that much of the complexity of teaching and learning mathematics problem solving results from the interconnections the learner must make among his or her mathematics resources, heuristics, control mechanisms, beliefs the learner has about problem solving, and a wide variety of affective factors. Many of the articles provide strategies for specific sections of the problem solving process (Dolan & Williamson, 1983; Krulick & Rudnick, 1989) but few look at the teaching of problem solving as a process. Dolan and Williamson (1983) suggest strategies for teaching six heuristics to improve mathematics problem solving performance in students. Each of these represents one stage in problem solving. Since problem solving is a much larger process, there needs to be an emphasis on the process and these heuristics should be taught in the context of the problem solving process.

Lester (1988) has been very interested in mathematics problem solving as a process and studying how students are able to control their learning. This metacognitive aspect of problem solving is also shared by Schoenfeld (1985) and Charles and Lester (1985). Schoenfeld (1985) talks about a "managerial strategy" which helps in the selection of the appropriate plan for the problem solution. To study the teaching and measurement of mathematics problem solving Schoenfeld (1982) conducted a study involving first- and second-year college students. There was a control (sample of eight students) and an experimental group (sample of 11 students). Each group was given a pretest (five questions) at the first class of the winter term and a posttest (five questions) at the end of the term. The experimental group took a one-month intensive problem solving course and the control group took another course designed to teach a structured approach to solving problems.

In each session there was small-group problem solving and whole-group problem solving. Schoenfeld modelled the problem solving process using problems in class. When he would arrive at a critical point in the process he would raise three or four options and evaluate them. After this evaluation he would pursue one of the options and then check back in a short time to see if the option was "working". By verbalizing his thought processes Schoenfeld was modelling the "managerial strategy". He also taught the students some commonly taught heuristics and put them in the context of the overall model for solving problems in class. This method makes use of metacognitive strategies. With the modelling the students developed a cognitive awareness of the whole process of problem solving. When the student was aware of the process then this "managerial strategy" guided the cognitive skills necessary to solve the problem. To assist students with issues of strategy (control) Schoenfeld had three main questions posted in the classroom. As they worked on the problems he would ask individual students to answer them:

1. What (exactly) are you doing? (Can you describe it precisely?)

2. Why are you doing it? (How does it fit into the solution?)

3. How does it help you? (What will you do with the outcome when you obtain it?)

At the beginning of the course the students could not answer these questions but at the end of the course they were asking and answering the questions themselves. This provided a strategy for the students to control the process as they worked through the problems.

The results showed that the control group had nearly identical scores on the pretest and the posttest. The experimental group had higher scores on the posttests than the pretests. Schoenfeld concluded that this was because the students in the experimental group had learned to use certain problem solving heuristics with some efficiency. However, there are a few factors that may have influenced the outcome of this study. The sample size was very small which may limit the generalizability of the results obtained. The control group was taught by a different instructor, which could have been a factor in the difference noted between the two groups on the posttest. Also, the knowledge component of the two courses was different, which could have influenced the outcome. The motivation of the students in the experimental group could also have affected the results. Schoenfeld actually taught the experimental group and was very enthusiastic about problem solving.

Charles and Lester (1985) investigated the effectiveness of a process-orientated program with grade 5 and 7 students in developing mathematics problem solving ability. There were 36 schools involved. At grade 5 there were 451 students with 23 teachers, and 485 students with

23 teachers at grade 7. Twelve treatment and 11 control classes at grade 5, and 10 treatments and 13 control classes at grade 7 were selected for the study.

The treatment used in the study focussed attention on each phase of Pólya's (1957) four-step model and the development of students' abilities to select and use a variety of strategies. This was accomplished by providing students with considerable experience working with process problems.

Four testing instruments (grade 5 and grade 7) with four problems each were developed by three mathematics educators before the study. These provided the testing material for the pretests, posttests, and the two intermediate tests for the treatment groups. Each student was scored in three areas for each test: understanding of the problem, the use of strategies in the planning stage, and the result of the work completed on the problem. A score of 0, 1, or 2 was given for each of these three areas for each problem. A score of 0 in the first area indicated no understanding of the problem, or inappropriate planning, and incorrect results: a score of 1 indicated partial understanding, planning, and results; a score of 2 indicated good understanding and planning and a complete result. The scores for the two complex translation problems (multiple step problems) were combined and the scores for the two simple translation problems (one step problems) were combined to give six scores for each test for each student.

At the beginning of the study a pretest was given to each student. Following the 23-week treatment both the treatment and control groups were given posttests. The treatment groups were also given a test at the end of 8 weeks and again at the end of 16 weeks to examine when the changes in problem solving ability took place.

The results of the study indicated that the treatment did improve the students' problem solving ability. There was improved performance in the understanding of problems and planning of solutions for problems. Improvement in obtaining correct results did not improve as quickly as the other skills. This indicates that there is much more involved in problem solving than learning the skills. The complex nature of knowing when to apply these skills appears to take more time to develop within the students. Charles and Lester (1985) mention the metacognitive nature of problem solving and the need to examine the influence of instruction on students' metacognition. This "managerial strategy" which Schoenfeld (1985) refers to may be operating to direct students' choices across the many commonly taught heuristics available for solving problems.

Lester (1988) mentions a teaching strategy for teaching problem solving that makes use of metacognitive strategies. This method has five features:

1. Instruction must take place in the context of learning a specific content.

2. The students' attention is focussed on solving a specific problem.

3. Students are allowed to make errors in their attempts.

4. Teachers are allowed to make errors.

5. Teacher's role is to guide and model. This will decrease as the students become more proficient at solving problems.

The role of the teacher in the problem solving classroom is multifaceted in nature. The teacher is an external monitor, a facilitator, and a model for the students (Lester, 1982) as the students engage in problem solving in class. In the teaching of strategies Popp (1993) points out that the teacher needs to plan lessons that follow the same general steps as any other instructional episode (e.g., the generic instructional methodology) (see Appendix A). This methodology has five steps: identify the learning, plan, interact, apply, and evaluate. Initially the teacher assists the students in moving to a stage at which they follow a visual plan for solving problems. Then the teacher assists the students to progress to independent use of the strategy.

This method of instruction for problem solving gives students support and yet allows them the opportunity to grow in their problem solving ability. The monitoring of the problem solving process is initially provided by the teacher. As the level of students' competence in problem solving increases the monitoring is taken over by the students.

Evaluation of Problem Solving

Evaluation is an integral part of all instruction. It is used to determine the effectiveness of the instruction, to provide an indication of the needs of the students, and to give an indication of the students' achievement. The complexity of mathematics problem solving has made evaluation very difficult. Most measurements of mathematics problem solving have used product measures rather than process measures. This brings up the question of validity of the measure. Are you actually measuring problem solving when you measure the product or are you measuring something else? When only the product is measured only one part of the problem solving process is being evaluated. However, most tests concentrate more on the correctness of the answer rather than on the procedures the students use. The evaluation needs to be much broader than a single score on a test if it is to give teachers enough information about the problem solving level of a student and provide information about instruction. Looking at problem solving as a single score does not promote the proper assessment of problem solving performance because it provides little if any information about the various stages of the process to guide future instruction.

Evaluation of problem solving in mathematics has been investigated using a few different methods (Charles et al., 1987; Malone et al., 1980; Schoenfeld, 1982; Silver & Kilpatrick, 1988; Veevers, 1992). In the United States the California Assessment Program Survey of Basic Skills (CAP) uses multiple choice questions to test mathematics problem solving performance. Silver and Kilpatrick (1988) believe that CAP is able to test some of the most important aspects of problem solving. However, the use of a multiple choice format with questions scattered throughout a test fragments the process and does not give students the whole picture of an integrated, active process. This format also allows the students to guess and obtain a correct result. The use of multiple choice, single score tests does not provide enough information for instructional purposes and does not allow the students to display the overall problem solving process.

Malone et al. (1980) developed a method to measure mathematics problem solving that attempted to address the complex nature of problem solving by providing more than a single score. This was one of the few methods that attempted to measure the process of problem solving. It gave a score for each section of the problem solving model and included some information provided by the students about their attempts.

First a list of nonroutine problems (these are problems that have no apparent solution at the onset) to be used in the test was developed. Each problem must take into account the level of the students based upon the mathematical ability of the students, the strategies required to solve it, the reading level of the students, and the length of the problems.

Once the problems were selected they were calibrated by having at least 150 students attempt each of the problems in the list. All problems which over 90% or under 10% of the students were successful in solving were removed from the list. Each question was scored by three markers using a scoring key (a five-point scale) which was provided. This new list of problems was then used to measure the students' mathematics problem solving ability. During the test the students were asked to record all of the details about their problem solving attempts. This scoring appeared to be highly subjective and could be open to different interpretations of the scale. However, it was found that there was a high degree of consistency among markers.

Schoenfeld (1982) developed an instrument for the measurement of problem solving performance consisting of three parts: the multiple count scoring, the "best approach" scoring, and the student assessment. In both the pretest and posttest there were five questions given to each of the two groups. The students were given the same instructions before the tests. They were told to use pen and to write down things they tried that did not work, different approaches to the problem solution, and reasons for doing what they were doing. Students in the intermediate division would require more instruction in this section before they could respond in a way that would benefit instruction.

Initially in the multiple count scoring, a list was made of the feasible approaches attempted by at least one student. Then the following questions were asked about each feasible approach.

1. Does the student show any evidence of being aware of this particular approach to the problem ("evidence")?

2. Does the student follow up on the approach ("pursuit")?

3. How much progress does the student make towards a solution ("progress")?

(a) Little or none ("little").

(b) A reasonable amount but not enough to claim the solution is almost complete ("some").

(c) Something close to a solution, maybe only an inaccurate calculation ("almost").

(d) A complete solution ("solved").

Each category would be given a 0 or 1. This scale seems to be highly subjective and to complete it with a high degree of consistency would require a great deal of training of the markers involved. However, Schoenfeld found that there was a high degree of consistency (over 90%) among the three markers.

Since these scores were nonordinal they would be difficult to rank or interpret. This would be a disadvantage since the teacher needs to have appropriate information to use in instruction modification and for determining levels of problem solving ability.

In the "best approach" scoring, each approach that the student used was scored separately. If an approach or plan was not pursued by the student the student received a score of 0. If it was considered "little" the score could be from 1 to 5 and up to 16 to 20 if it was considered "solved". The best effort for each problem dictated the final score.

The third score was a measure of the students' reactions to their performance in each of the five problems in the pretest and posttest. They were given 4 minutes after each problem to answer some questions relating to their reactions. This section could give some valuable information about the students' feelings but very little information about the thinking process.

Charles et al. (1987) describe three methods for evaluating mathematics problem solving: observing and interviewing students, student self-assessment, and holistic scoring. Each has advantages and disadvantages that need to be considered. The observation and interviewing of students is useful for small groups and can provide a great deal of information but is very time consuming. Students are asked to respond to a number of questions in a one-to-one situation. The instructor also needs to be quite knowledgeable about problem solving to understand the data collected. The second technique requires the students to report on the problem solving situation they have completed. This can provide useful information but is limited by the ability of the student to communicate his/her solution. In the third technique the teacher assigns a point value to each stage of the problem solving process. First the stages must be decided upon and then the point value for each stage is described. Finally the criteria need to be developed for each point value to enable the teacher to assign the points. This method is difficult to score but can give an idea of the areas of weakness in the process of problem solving. These methods could be useful in the classroom and if carefully constructed could yield valuable information about the problem solving ability of students.

Another method used to evaluate a student's problem solving ability yields a great deal of information for the teacher and the student but was quite lengthy and time consuming. This technique is a step-by-step process where the student completed one step in the logical numerical problem solving strategy before moving on to the next stage (Eagan, 1993). When the student completes the first step in the strategy, he or she goes on to the next stage. However, the student is not allowed to turn back to a previous step any time. The student is allowed to go to the next step only when the present step is completed. This would provide the teacher with information about the stage in which the student was experiencing difficulty. An adaptation of this approach and parts of the methods described in Charles et al. (1987) were utilized and expanded upon for this study.

The research results of student behaviour during problem solving (process) also need to be examined carefully to develop effective assessment techniques. This information could then be used in designing or selecting assessment strategies.

Nature of Problem Solving in Mathematics

In 1910 it was clear from Dewey's work (cited in Stanic & Kilpatrick, 1988) that problems and problem solving were crucial to his view of education. In 1988 the National Council of Supervisors of

Mathematics stated that "learning to solve problems is the principal reason for studying mathematics" (p. 2). One year later in 1989 the National Council of Teachers of Mathematics, in their document, Curriculum and Evaluation Standards for School Mathematics, emphasized that "problem solving must be the central focus of school mathematics" (p. 6). In 1993 the OAME and the OMCA, in the document, Focus on Renewal of Mathematics Education: Guiding Principles for the Early, Formative and Transition Years, stated that "problem solving is an integral part of instruction" (p. 17). Even with these insights and directions problem solving has not become an integral part of mathematics education.

The subcommittee of the Ontario Association of Supervision and Curriculum Development (OASCD) made recommendations to the Benchmarks Committee of the Ministry of Education in 1992 concerning mathematics education. The subcommittee developed eight fundamental principles of mathematics education for grades one through nine. The eight principles are provided to show the flavour of these recommendations:

1. The process of mathematics consists of choosing models; of studying, analyzing and manipulating them, and of relating the findings to the real world" (Ontario Ministry of Education, 1975, p. 61).

2. To be authentic, the process of mathematics must be applied to the students' ongoing "world" interests.
3. The strategies involved in constructing and employing models are developmental in nature, moving towards states of higher sophistication, effectiveness and efficiency.
4. The construction and use of models is an active process - involving problem solving, creativity, and estimation - that is facilitated by the use of manipulatives and materials that enable concrete representations.
5. The use of models requires students to see patterns between the values of model variables - patterns that provide the basis for making generalizations and predictions about phenomena in the real world.
6. Model building and use provides opportunities for both independent and collaborative learning strategies.
7. The "real world action" conclusions reached by building and applying models should be presented in coherent written or verbal arguments that outline the model employed, the manipulations made on problem elements, and the results of these manipulations.
8. The model building and use cycle should constitute the central

continuing core of the mathematics program, to which all other aspects are subordinated. (OASCD, 1992, unp.)

Model building is central to the mathematics program and the problems are the context within which the model building takes place. Logic is the tool used to build these models. Problem solving, which is a central part of model building, should be a central part of mathematics education.

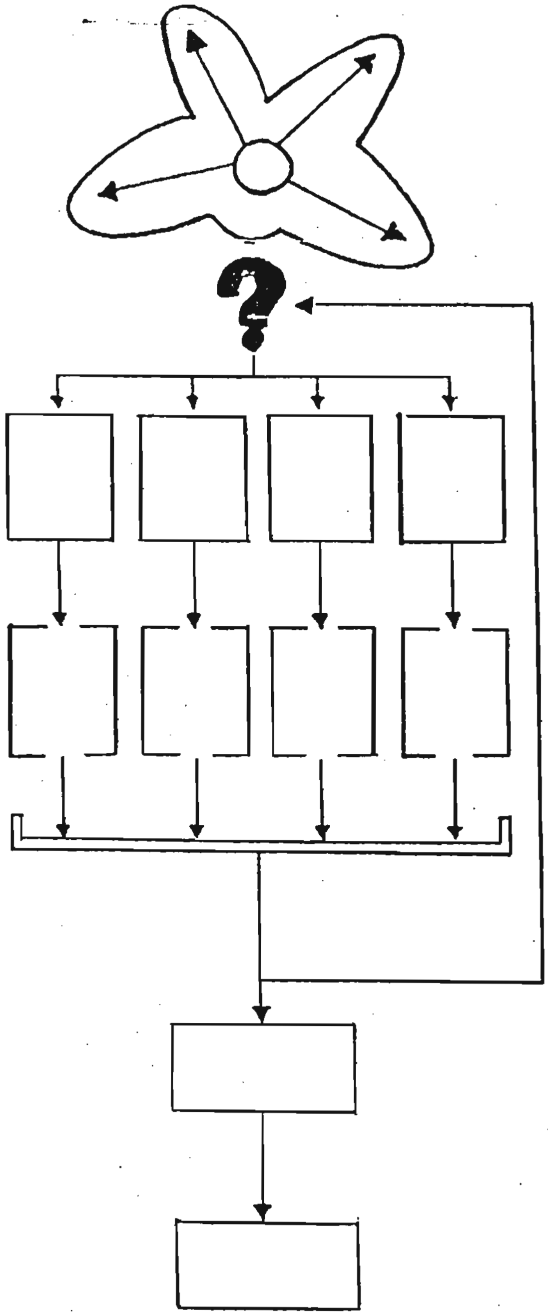
In 1995 The Common Curriculum: Polices and outcomes, grades 1-9 developed by the Ontario Ministry of Education and Training stated that "learning is an active process...curriculum should encourage this kind of constant inquiry"(p. 17) and "the processes of inquiry and problem solving are basic to the study of mathematics, science, and technology" (p. 79). The OAME and the OMCA (1993) recommended that students should be working with and creating problems from real-world activities and solving them in mathematics classrooms. The need for relevant and real-life problem solving in mathematics education is critical for developing problem solving abilities of students.

Generic Problem Solving Strategy

For educators to facilitate the growth of problem solving in each student there is a need for a model to clearly define the process. Prior to the seventies problem solving was viewed as the application of a specific algorithm to a problem to arrive at the correct answer. This involved the students being engaged in the application of the new skill, concept, or algorithm to a situation presented within each chapter in the textbook. The majority of textbooks have been careful to maintain this regular course of problem solving.

In 1975 Popp, Robinson, and Robinson developed a generic problem solving strategy (see Figure 1). The strategy has five main steps but was expanded to seven to be more applicable to the classroom situation. It is the model currently used in many Ontario classrooms.

The generic problem solving strategy identifies the major steps an individual proceeds through in problem solving. When a problem situation is confronted there is some time required to explore the situation. This exploration stage is important to formulate the question that will be the focus for the problem solving. Once this is established, the individual generates many alternatives (plans) to solve the problem generated in step two, or identifies a number of factors to pursue to resolve it. To select the best alternative there needs to be some information (data) used to check each alternative. These data can be from the problem or from previous knowledge acquired by the individual, or they can be searched out or



EXPLORATION: Introducing exploratory activities.

INQUIRY QUESTION: The student poses a suitable question around which the study will develop.

FACTORS: The student suggests a range of reasonable alternatives to answer the question. (Additional alternatives may arise in the subsequent data collection phase.)

DATA COLLECTION: The student collects information on each factor.

SYNTHESIS/CONCLUSION: The student arrives at a conclusion by deciding, on the basis of the accumulated information, which of the alternatives give(s) the best answer to the question.

COMMUNICATION: Organize a clear expression and presentation of the conclusion.

EVALUATION: Assess the appropriateness of the conclusion and its expressionin light of the original question.

Figure 1. The basic inquiry model (reproduced with permission from Popp Robinson, and Robinson, 1975).

generated. This gives the individual the information to select the alternative which represents the best solution for the problem. Then the conclusion is organized in some way to give a clear and concise indication of the solution. Finally the solution is checked by the individual to see if it adequately answers the question.

In his writing, Dewey (1910) referred to problem solving as reflective thinking which is exactly what happens in this generic model. If the model were represented graphically, it would have many interconnecting lines. It is drawn in a linear fashion for easier explanation and to show the overall flow (see Figure 1).

The study of problem solving begins with the model for reflective thinking by Dewey (1910). Dewey used the phrase "reflective thinking" instead of "problem solving" in his book, How We Think (1910, 1933). Through a number of situations described in his book, Dewey (1910) outlined his five-step model:

1. Difficulty experienced.
2. Thorough exploration of the problem.
3. Suggestion of a possible solution (called heuristics).
4. Development of the suggestion.
5. Verification of the suggestion - conclusion.

In 1957 George Pólya outlined a four-step model for problem solving. It contained elements of Dewey's model and combined steps one and two into one step. Pólya outlined the following four steps in his book, How to Solve It (1957) as a model for problem solving:

1. Understand the Problem.
2. Devise a plan.
3. Carry out the plan.
4. Look back.

This is a sound model and is the basis for the five-step model proposed by the OAME and the OMCA in Focus on Renewal of Mathematics Education: Guiding Principles for the Early, Formative and Transition Years (1993). This model adds a fifth step to Pólya's model which is communicating the solution. However, the generic problem solving strategy expands further upon this model and provides a useful classroom model for teachers and students (see Figure 1).

The first step in Pólya's model is extremely important but it does not give enough direction about understanding the problem. How do we understand the problem? Popp and Seim (1978) explain that you need to explore and question. This is more specific and helps to define and guide exactly what should be done. The second step in Pólya's model is to devise a plan. Again this step needs to be elaborated to be useful within the classroom. This step is expanded into three steps (three, four, and five) in the generic problem solving strategy. The synthesis stage in the generic problem solving strategy is part of Pólya's steps two and three. The generic problem solving strategy goes on to include the expression of the conclusion and evaluation of the process that is part of Pólya's fourth step. The generic problem solving strategy model has expanded upon Pólya's ideas and provides a more detailed plan to be utilized in the classroom.

Problem Solving in Mathematics

In the development of a strategy there is the problem of being too general or too specific. If the strategy is too broad, trying to include all problem types, it would not be convenient to specific problems. If it is so specific that it pertains to only one type, then its use is very limited. The generic problem solving strategy is a model that is widely applicable across disciplines. Since problems in the school disciplines differ in context there is a need for adaptations of the generic problem solving strategy for use in the different disciplines. Mathematics is an example of such a discipline in which generation of a specific model would make problem solving more applicable and efficient.

Mathematics is an area in which it has always been said that problem solving was being “looked after”. The assumption was that there was problem solving in the classrooms. According to the textbook problems, which traditionally has been the way problem solving has been handled in the classroom, it appears that only exercises have been worked on and the process of problem solving has not been addressed fully.

Dolan and Williamson’s book (1983) looks at six heuristics (guess and check, make a table, patterns, make a model, elimination, and simplify) and how to teach and use them in the classroom. The students need to recognize that these suggestions represent only a part and not the whole process of problem solving. Problem solving in mathematics is a process that requires an overall model into which such heuristics can be embedded.

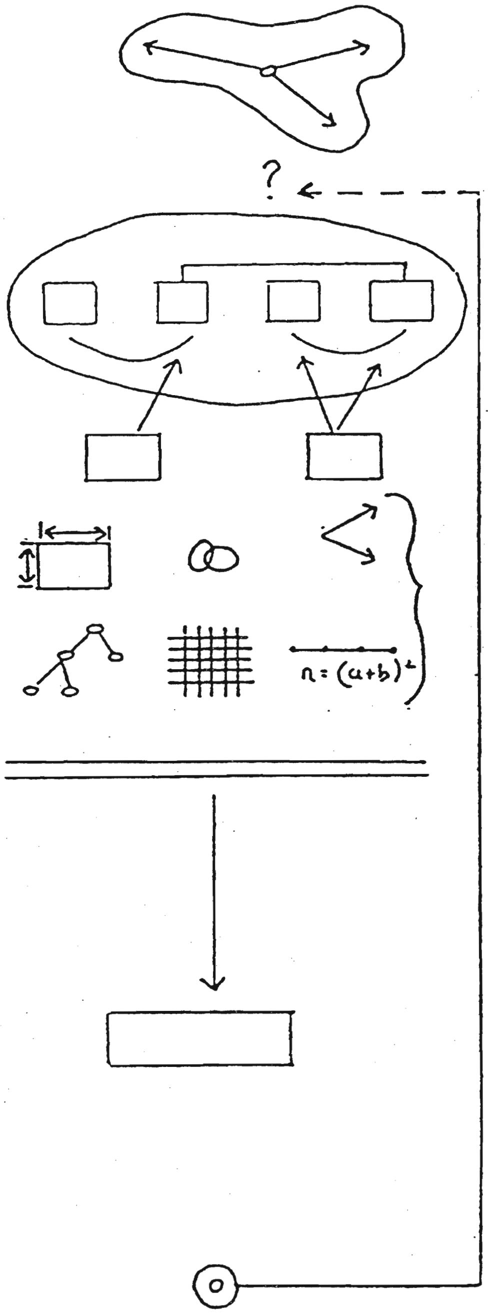
These are the commonly taught heuristics in mathematics: guess and check, find a pattern, think of similar problems, simplify, restate problem in your own words, make a diagram, chart or table, work backwards, and decide on relevant and irrelevant information (Chadwick, 1984; LeBlanc, 1977; Pólya, 1957; Schoenfeld, 1980). If these are examined in relation to the general problem solving strategy one would discover that each heuristic is an operation performed at some steps of this model. By focussing on these as different approaches to problem solving instruction, attention has not been directed to the model within which these are embedded.

A second implication of the deficiency between the generic problem solving strategy and the use of these traditional heuristics in problem solving in mathematics is that it helps to hide a useful relationship. By consciously embedding the suggested operations in the generic problem solving strategy, the problem solving can be made more specifically applicable to the type of problem solving normally engaged in within the mathematics classroom.

Making the generic problem solving strategy more acceptable to the mathematics teachers will provide them with a basis for more effective instruction of mathematics problem solving as a process.

The mathematics problem solving strategy model is an adaptation from the generic problem solving strategy (Popp, Robinson & Robinson, 1975). This model is applicable to real life situations in which numbers can be used within the problem solving process. It is called the logical numerical problem solving strategy (see Figure 2).

There are many similarities between the logical numerical problem solving strategy (LNPSS) and the generic problem solving strategy. The diagrammatic representation of the two models (see Figure 3) shows the relationships between the steps. In both of the models the need for exploration and investigation of the problem situation is a critical beginning stage. Following this investigation a focus question is



1. Problem: Investigate the context, to establish the general nature of the problem.

2. Question: Identify the essential focus (the "unknown").

3. Organization
   (a) Analysis: Identify the essential elements and their relationships;
   (b) Information: Identify given information;
   Recall or locate other relevant information;
   Identify applicable relationships.

   (c) Representation: Select or design appropriate model or representation;
   Translate into manipulable form.

4. Calculation: Select appropriate algorithm(s);
   Sequence steps;
   Calculate.

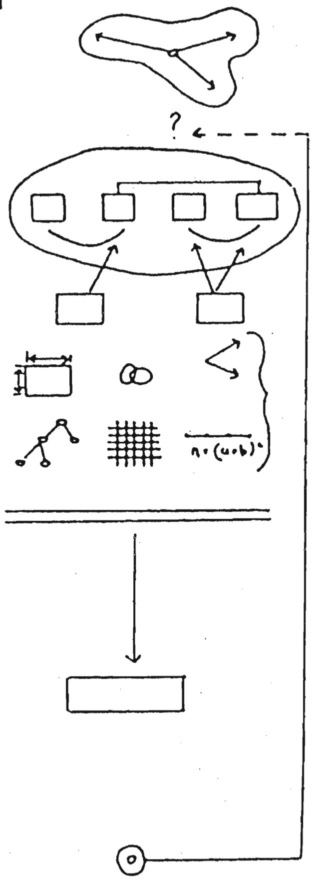
5. Conclusion: Interpret the result of the calculation.

6. Record: Make a record of procedure and/or results, as appropriate.

7. Evaluation: Assess for:
   Reasonableness of conclusion;
   Accuracy of calculation;
   Efficiency of organization;
   Generalizability of conclusion and organization;
   Sufficiency and clarity of record.

Figure 2. Logical numerical problem solving strategy (reprinted with permission from Popp and Seim, 1974).

Logical-Numerical Problem-Solving Strategy



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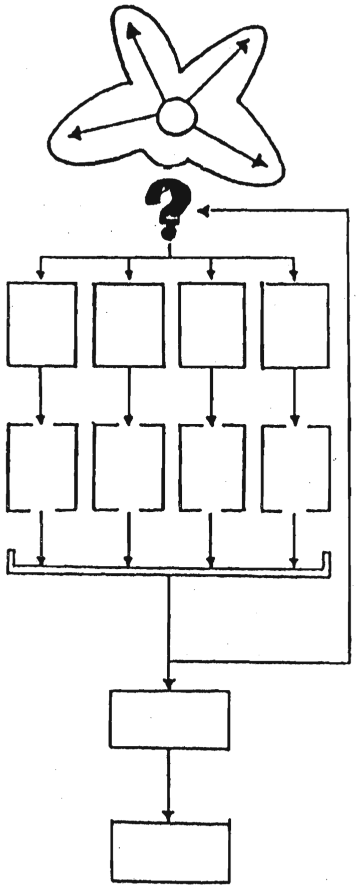
THE BASIC INQUIRY MODEL



EXPLORATION: Introducing exploratory activities.

INQUIRY QUESTION: The student poses a suitable question around which the study will develop.

FACTORS: The student suggests a range of reasonable alternatives to answer the question. (Additional alternatives may arise in the subsequent data collection phase.)

DATA COLLECTION: The student collects information on each factor.

SYNTHESIS/CONCLUSION: The student arrives at a conclusion by deciding, on the basis of the accumulated information, which of the alternatives give(s) the best answer to the question.

COMMUNICATION: Organize a clear expression and presentation of the conclusion.

EVALUATION: Assess the appropriateness of the conclusion and its expressionin light of the original question.

Figure 3. Comparison of basic inquiry model and the logical numerical problem solving strategy.

established. Given this focus or question there are many factors that are considered and are to be organized in some manner. In the LNPSS, the major emphasis is on the identification of relevant factors, and on effective ways of representing the relationships among the factors. The synthesis stage in the LNPSS can involve one or more calculations. In both models there is a conclusion and then a method of recording and communicating this result. Finally in both strategies there is an evaluation stage in which the steps and process are evaluated by the problem solver.

The LNPSS should be utilized to instruct students in mathematics problem solving since it relates directly to mathematics. This must be based upon a cumulative learning approach to realize growth in students' problem solving ability. Popp and Seim (1974) reported three requirements for this approach:

1. The teacher must have access to a growth plan that identifies the stages through which student learning will progress as the student works to achieve the mature performance required by society.

2. The teacher must compare student performance at any point in time against the growth plan to identify those stages that the student has already mastered. The teacher must have both the time and ability for this diagnostic procedure.

3. The teacher requires the instructional skill to design learning situations that will promote student growth from any stage in the growth plan to a higher level of performance.

Once the program is implemented there is a need to assess the growth in each student's problem solving ability. This can be realized by using growth strands developed by the author for the LNPSS and adapted from growth strands from Borthwick and Fowler (1989) for the generic problem solving strategy.

Summary

This chapter has examined the nature of problem solving in mathematics, the instruction of problem solving in mathematics for growth, and the evaluation of problem solving performance. Research about mathematics problem solving has focussed on increasing our understanding of problem solving as a process. However, there are still relatively few research studies about the instruction and evaluation of mathematics problem solving. Many of these evaluation instruments that are available have used the product rather than the process as a measure of problem solving ability. There are a few that attempt to measure the process of problem solving (Charles et al., 1987; Malone et al., 1980;

Schoenfeld, 1982) and one very promising measure which utilizes a step-by-step approach (Eagan, 1993) using the LNPSS.

The "managerial strategy" as described by Schoenfeld (1982) appears to be a promising metacognitive strategy for teaching the process of mathematics problem solving utilizing many heuristics.

The LNPSS provides a model for mathematics problem solving instruction in which the commonly used heuristics can be embedded. This could be used for instruction and evaluation of problem solving.

CHAPTER THREE: METHODOLOGY AND PROCEDURES

Overview

This study was based upon an experimental design using whole classes of students in an actual school setting. The materials for instructional purposes were designed by the author. The instrument for assessing mathematics problem solving was adapted from an instrument developed by Eagan (1993) and Charles et al. (1987).

The study has three main parts: the assessment of mathematics problem solving ability, the evaluation of growth in this ability using growth strands developed for the LNPSS, and the teaching of a strategy for mathematics problem solving. For all parts the subjects involved in the sample were selected on a nonvoluntary basis. The parents were provided with information about the project prior to the beginning of the study. The dependent variable of problem solving ability was measured using pretests, posttests, and postposttests (see Appendix B). Also, observations of student behaviour during problem solving sessions were collected.

Hypotheses

This study involved the teaching of a strategy for mathematics problem solving and the use of growth strands, developed for the LNPSS, for the measurement of mathematics problem solving ability in students. The problem statements have generated three null hypotheses:

1. Students in the two classes will not vary significantly in their problem solving ability range prior to the initiation of the study.

2. The teaching of a strategy for mathematics problem solving will not significantly improve the students' ability to solve problems in mathematics.

3. It will not be possible to assess growth in problem solving performance using series of observable steps for each stage of the logical numerical problem solving strategy (growth strands).

Research Design

The study was a quasi-experimental design using pretest, posttest, postposttest and a control group. The two class groups involved in the study were assigned at random to either the experimental group or the control group. All subjects received a pretest to determine the level of problem solving ability.

The treatment, which consisted of teaching them the LNPSS as part of the daily lesson, lasted 6 weeks. The control group received the regular unit instruction for 6 weeks. After this time both groups received a posttest. Following this, the control group received the treatment for 6 weeks while the treatment group received the regular unit instruction. At the end of this time the control group (new treatment group) received a postposttest.

There were three problems used in the evaluation instrument. In the pretest all students worked individually with the same problem. For the posttest there were two problems. The classes were divided in half and each half did a different problem. In the postposttest the same problems as the posttest were used with students working with the problem different from the one in the posttest.

Growth strands were developed by the author for the LNPSS (see Appendix C). These were adapted using the growth strands developed by Borthwick and Fowler (1989) and Popp (1986) for the generic problem solving strategy.

Background and Selection of Subjects

The site of the study was a secondary school in a southern Ontario community of approximately 4,800 residents. The school opened in September 1992 with a student population during the 1992/93 school year of approximately 525. The student population for the 1993/94 school year was 575. Most of these students came from the urban setting (60%) with the remainder (40%) from the surrounding rural area.

The subjects that made up the sample were grade 9 students. There was no screening of the subjects before its beginning. Two full classes of students were selected, a total of 43 students. One class had 20 students and the other had 23. The students' parents received information outlining the study prior to the beginning of the study. There were 45% rural and 55% urban represented in the sample. There was also a large percentage of Native Canadians in the sample (24%). The sample consisted of 17 females and 26 males, ranging in age from 12 years 9 months to 15 years 5 months.

Pilot

Prior to the study the interrater reliability of the testing instrument for the problem solving ability level of students was investigated. Two secondary school mathematics teachers, each with at least 10 years of teaching experience in teaching mathematics were selected on a voluntary basis as markers. The assessment instrument was administered to four grade 9 students by the author. Following this, the tests were divided between the two markers who were given directions and a key for scoring the tests. Each test had only a code for identification. The results were analyzed for between-marker differences.

Description of Research Methodology

The experimental treatment involved mathematics problem solving instruction with the teacher following the LNPSS. This consisted of a 37-minute lesson each day for 6 weeks (a total of 30 sessions). The comparison treatment, which the control group followed, involved a mathematics unit on problem solving with no instruction with a problem solving model.

Treatment

Following the mathematics problem solving ability pretest, the students in the experimental group began receiving instruction in the LNPSS. This occurred at the same time each day over the 6 weeks.

Classroom Procedures

The lesson format followed the same pattern each day. The treatment group would begin the lesson by reviewing the problems from the day before. Then there would be time for development of the LNPSS led by the teacher, and finally a time to apply the strategy to real problems. The introduction and development of the LNPSS to the level of independence was realized using the generic methodology (see Appendix A) emphasizing the metacognitive aspects of problem solving during the modelling of the process in class.

Preplanning for Day One

The objective or learning for the first lesson after the pretest was to have the students follow a visual plan for solving problems and to use the LNPSS as the plan.

Prior to the lesson the teacher had to identify the present level of ability, which came from the pretest and from the students' memories of solving problems and what their experiences were during that process.

The teacher then needed to identify the level to be attained by the students. For this introductory lesson it was to have the students follow a self-generated visual plan to solve problems. The nature of the transition from the present to the intended level involved reflective analysis by the students.

Day One

In the lesson the teacher introduced a preselected problem to the class. Experiences of the teacher and the students were cued by the teacher to serve as a guide in the process. Students then reconstructed three previous problem solving experiences and identified the commonalities between the strategy in class and their own strategies. These commonalities were labelled by the students and they applied them to a current problem selected by the teacher. Next the students applied their plan to new problems in class and took them home if they were not completed. During this lesson and all subsequent lessons a diagram of the LNPSS was displayed on the blackboard.

Preplanning for Day Two

The objective for this day was to develop independent use of the LNPSS. The teacher determined the present level of ability of the students, from the previous day's activities. The level of functioning of the students was the ability to follow the LNPSS using cues and steps from the blackboard. The intended level was the independent use of this model. The nature of the transition from the present to the intended level was procedural facilitation that utilized the fading of cues.

Day Two

First the problems were discussed and questions were clarified. The teacher and students then selected a problem from a list provided by the teacher. The teacher used the diagram of the LNPSS on the blackboard and verbally cued the students as they progressed through the model. Next the diagram was used with a student doing the cueing and finally the diagram was used with self-cueing. Most students could reach a stage without reference to the diagram at all. However, the diagram remained on the blackboard for the entire study. At the end of lesson the model was applied to a new problem.

Finally the students were given a problem to do in which they were to teach the model to someone at home and discuss the steps. After 2 days each student reported on what he/she had discovered.

Typical Day

Following the first 2 days a daily pattern for the class time was established which was followed each day during the treatment. The teacher focussed on two learnings for the students: learning how to use the selected heuristic within the model and learning when it was appropriate to use the heuristic. Each day began with a review of the homework problems as a whole class discussing and clarifying difficulties. Following this a heuristic (see Appendix D), to be embedded within the LNPSS, was introduced using a problem preselected by the teacher. The teacher led the students through the problem on the blackboard. As the teacher progressed through the problem she/he verbalized what he/she was thinking. The teacher also asked questions to clarify points during the process. Students were encouraged to ask and answer questions and to assist in decision making during this time.

During the next part of the lesson the students were placed in groups (four or five per group) to work through two to three assigned problems. The teacher facilitated the interaction within each group that utilized the new heuristic within the LNPSS. At the end of the period the groups took these problems home to complete. These homework assignments were reviewed in class the next day and the lesson pattern was repeated. It should be pointed out that the interests of the students were taken into account when the problems were developed. Some examples of the different problems used to teach the strategy are provided in Appendix E.

Typical Day After All Heuristics Were Introduced

Each day the problems were discussed to clarify any difficulties. Following this a new problem would be generated by the teacher and sometimes the students. Once the problem situation was established time was provided in small groups (four or five students) to determine the focus or question of the problem. Then together the teacher and students worked through the problem following the LNPSS and applying specific heuristics as required. The teacher would model the behaviour and ask questions during the process to demonstrate what he/she was thinking. Next the teacher provided time for the students to work with one or two problems applying the knowledge about the LNPSS.

Control Group

The control group was taught a unit using powers and square roots, a regular part of the grade 9 curriculum, using problem solving as the focus. There was no instruction on the LNPSS or the use of heuristics during these sessions. This was implemented with whole-class, group, and individual assignments. Following the 6-week period the control group received the treatment for 6 weeks and the experimental group received the regular grade 9 curriculum as described for the control group.

Growth Stranding

The growth strands for the LNPSS were developed by the author by adapting the growth strands developed by Borthwick and Fowler (1989) for the generic problem solving strategy. The work by Popp (1986) and Pólya (1957) was also a source of information in the formulation of these growth strands (see Appendix C).

Each level within the growth strands for each stage in the LNPSS was assigned a numerical value. This was used to calculate a score for each stage of the problem solving strategy for each student.

Instrumentation

Step-by-Step Approach

This is a very time consuming measure that yields great amounts of useful information. This approach was used for the pretest, posttest, and the postposttest (see Appendix A). The test consisted of the LNPSS divided into seven stages on separate sheets of paper. It was completed in one period and collected at the end of the session.

The students were asked to complete the first section and to write down as much as they could about what they did and why. Then they were to go on to the next section and not look back to the previous page or to go forward. On this next section a correct response was provided for the previous stage. This was done to ensure that the students were being evaluated for the stage that they were working on, uncontaminated by performance on the preceding stage. They were to complete this stage of the problem and again write down everything they could about what they did and why. Once all stages were completed in this fashion, the papers were collected and taken to the markers for evaluation.

The markers used the growth strands (see Appendix C) to evaluate the stages in the model. Each student received a separate score for each stage. Tables were generated to record the results for the treatment and the control groups.

There were three problems developed for the testing instrument in this study. Each student worked on a different problem for each testing session. For the pretest all of the students worked on the same problem. The posttests, which contained two different problems, were given to the students to assess the growth in the students' ability to solve mathematics problems at the end of the 6-week treatment period. Each class was divided in half with each half completing one of the two problems. The postposttest had the same two problems as the posttest. The class was again divided in half and each student worked on the problem different from that in the posttest.

Other Measures

Each student's age was calculated as of June 1, 1993 using the school's official register for the two classes involved. Since student attendance was a possible bias in the results it was recorded from the school's official register.

Scoring, Data Collection, Recording, and Analysis

All data from the pilot test were gathered and analyzed using the Pearson Product-Moment Correlation Coefficient to test for differences between the markers ($p \leq 0.05$). In an attempt to eliminate marker bias all pretests, posttests and postposttests were coded. The markers saw only a letter code for each test.

Scoring

The scorers were trained in the use of the scoring instrument to achieve a 90% agreement. At each stage of the LNPSS a value was allotted for each student according to the student's performance at that stage.

Data Collection

Each scorer scored one half of the papers. The scores for each stage in problem solving were tabulated to give a total score for the problem solving ability for each student. The information gathered from the pretests, posttests, and postposttests were entered into tables and graphs for analysis.

Recording

The modal level of the response at each stage of response was recorded in tables. Graphs were generated to provide a basis for analysis of the results. A graph with the pretest and posttest scores for both the control and the treatment groups as well as separate graphs for the pretest, posttest, and postposttest scores of the control and pretest and posttest scores for the treatment groups were constructed.

Analysis

Analyses of the differences between the pretests of the control and treatment groups were conducted utilizing t-tests for independent groups ($p \leq 0.05$). The posttest and postposttest scores for the control group were analyzed using paired t-tests ($p \leq 0.05$). Further to this a MANOVA table was completed using the pretest and posttest scores for the treatment and control groups to determine significance of differences across groups ($p \leq 0.05$).

Assumptions

There are a few assumptions made upon which this study was based.

1. The mathematics problem solving process is a teachable process.
2. All problems share three similarities (Popp & Seim, 1978):

(a) There is an identified difficulty, issue, or inconsistency for which there is no immediate resolution.

(b) There is the desire (or motivation) to engage in attempting to find a resolution.

(c) There is more than one reasonable solution or approach to solution available for consideration.

3. Problem solving in mathematics can be described as the LNPSS which is an adaptation of a generic problem solving strategy.

4. Since problem solving is a process, it is the process that needs to be evaluated as well as the product. It is difficult to evaluate these thinking processes which are involved in the process of problem solving.

5. The method for teaching problem solving depends on the teacher's definition of mathematics problem solving. If the teacher defines problem solving as the product, then the method of instruction will be very different from that of a teacher with a process definition.

6. There are identifiable behaviours which can be observed at each stage of the problem solving process. This is critical for the development and usefulness of growth strands.

7. There are measurable gains in problem solving ability which can be realized over a 6-week period.

Limitations

There are a number of limitations inherent in this study that would limit the generalizability of the conclusions. The sample size was 23 students in the control group and 20 in the experimental group. Also, all of the data were gathered from one secondary school in a rural board of education in southern Ontario. This may or may not be representative of the students across Ontario. The results can only be generalized to the extent that the sample is representative of any population of interest. Also the use of only one problem solving strategy (LNPSS) restricts the generalizability of the results.

The evaluation of problem solving ability is dependent upon the information provided by the student on the test. Therefore the amount of information received from the student on the pretest, posttest, postposttest was a limitation for this study. This study was conducted with these limitations in mind.

Summary

The purpose of this study was to measure the mathematics problem solving ability of students and the growth of this ability using growth strands developed for the LNPSS and to determine the effect of teaching a mathematics problem solving strategy on students' ability to solve problems in mathematics.

This chapter discussed the null hypotheses, research design, description of the sample, and procedural methods for this study. There were three hypotheses generated which looked at the range of problem solving ability, the effect of teaching a strategy (model), and evaluation of problem solving ability.

The research design used a treatment lasting for 6 weeks and a control group along with a pretest, posttest, and postposttest. The sample consisted of two grade 9 classes from a small rural secondary school in southern Ontario. Subjects were selected on a nonvoluntary basis with no screening prior to the study.

The pilot of the testing instrument was done to investigate between-marker differences. Two markers were used for the scoring of the papers in the study. The testing instrument used was a step-by-step procedure that utilized the growth strands for the LNPSS for evaluating each stage. This resulted in a score for each stage for each student. These scores were then analyzed using t-tests and a MANOVA ($p \leq 0.05$).

Careful attention was given to the implementation of the LNPSS in the classroom and detailed information was provided on the format of the lessons and daily routines used throughout the study. A number of limitations and assumptions were outlined which were kept in mind during the duration of the study.

CHAPTER FOUR: PRESENTATION AND ANALYSIS OF RESULTS

Overview

This chapter consists of two sections: presentation and description of the data, followed by interpretation of the results. The data will be presented in graphic as well as tabular form to allow for greater clarity in the descriptions.

Results

Initially there were 23 students in the control group and 20 in the treatment group. One student from the control group moved to another school and one was absent for 3 weeks of the 6-week treatment, leaving 21 students. The treatment group also had one student absent for 2 weeks leaving a sample size of 19.

Each of the tests in the study (pretest, posttest, and postposttest) generated eight scores including the total score for each student. The scores were clustered into three groups plus the total score for analysis: the exploration cluster was obtained by adding together the scores from the exploration and questioning stages of the LNPSS; the planning cluster was calculated by adding together the scores for the organization and calculation stages; and the conclusion/evaluation cluster was calculated by adding together the scores from the conclusion, record, and evaluation stages.

Results of the pilot study, using the Pearson Product-Moment Correlation Coefficient, generated interrater reliability estimates for the three cluster scores and the total score for the LNPSS. The coefficients for the two scorers for each of the clusters ranged from 0.84 for the exploration cluster to a high of 0.94 for the total score. All Pearson product-moment correlation coefficients were significant at $p \leq 0.01$ (Table 1).

A mean and standard deviation for the three clusters from the LNPSS were calculated for the control and treatment groups and are presented in Table 2. Comparison of the pretest means of the control and the treatment groups at the beginning of the study showed no significant difference (Table 3). Great variability in the scores within the two classes was evident from the high standard deviation (SD) in each group (Table 2). In the pretest two of the four SDs were lower in the treatment group. In the posttest cluster scores the SDs for the treatment group were lower in three of the four instances. However, of the four treatment SDs only one decreased (Exploration Cluster) in the posttest. All of the other SDs increased for both groups from the pretest to the posttest. The SDs from

Table 1

Pearson Product-Moment Correlation Coefficient for Two Scorers

Logical Numerical Problem Solving Strategy Stage	Reliability Estimate[n]
Exploration Cluster	0.84*
Planning Cluster	0.91*
Conclusion/Evaluation Cluster	0.89*
Total Test Score	0.94*

$n = 8$

* $p \leq 0.01$

Table 2

<u>Pretest, Posttest, and Postposttest Means and Standard Deviations for the Three Cluster Scores and Total Score of the Logical Numerical Problem Solving Strategy</u>

		Pretest				Posttest				Postposttest			
Group		1	2	3	Total	1	2	3	Total	1	2	3	Total
		(5)	(7)	(5)	(17)	(5)	(7)	(5)	(17)	(5)	(7)	(5)	(17)
Control^{n_a}	M	2.85	3.57	1.33	7.76	3.57	3.42	1.42	8.42	4.09	4.04	1.85	10.00
	SD	1.01	1.69	0.96	2.89	1.28	2.27	1.36	4.41	0.53	1.71	1.15	2.81
Treatment^{n_b}	M	2.68	3.73	1.84	8.26	3.64	4.15	2.05	9.85				
	SD	1.18	1.32	1.01	2.80	0.82	1.95	1.43	3.47				

1 = Exploration Cluster, 2 = Planning Cluster, 3 = Conclusion/Evaluation Cluster.

Numbers in parentheses are maximums for each measure.

$n_a = 21$

$n_b = 19$

Table 3

Probability Values for t-Test Results for the Pretests, in the Treatment and Control Groups and Posttests, and Postposttests in The Control Group

Groups (Test)		Control (Pre)				Control (Postposttest)			
Clusters		1	2	3	Total	1	2	3	Total
Treatment	1	0.61							
(Pre)	2		0.73						
	3			0.11					
	Total				0.58				
Control	1					0.08			
(Post)	2						0.23		
	3							0.03*	
	Total								0.05*

Note: 1 = Exploration Cluster, 2 = Planning Cluster, 3 = Conclusion/Evaluation Cluster

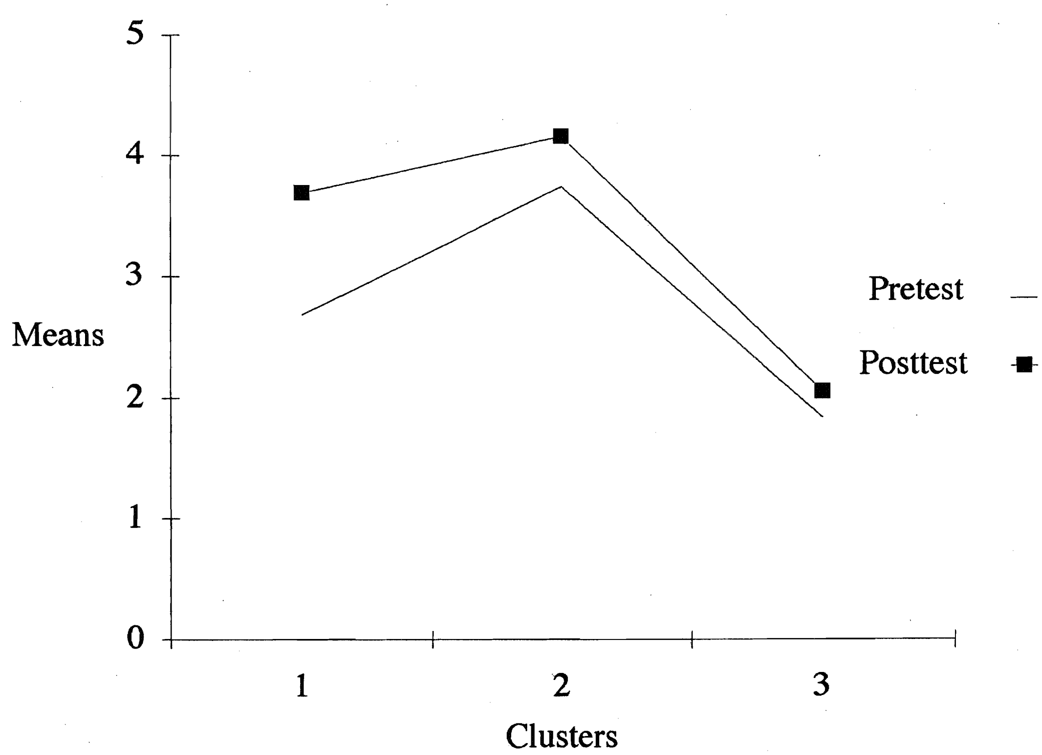
* $p \leq 0.05$

posttest to postposttest decreased in all three clusters and total scores. This demonstrated a decrease in the range of the problem solving achievement level of the students in this group.

All cluster scores and the total scores increased more in the treatment than the control group. This was consistent in all pretest and posttest comparisons (Figures 4 and 5). A percentage basis reveals a 35% increase in the exploration cluster in the treatment as compared with a 25% increase in the control group. Similar results were obtained in the conclusion/evaluation clusters (6.7% in the control and 11.4% in the treatment group) and the total scores (8.5% in the control and 19.2% in the treatment group). The greatest gains were made in the exploration cluster scores in the treatment group. Planning and conclusion/evaluation clusters increased by approximately the same percentage (11%) in the treatment group.

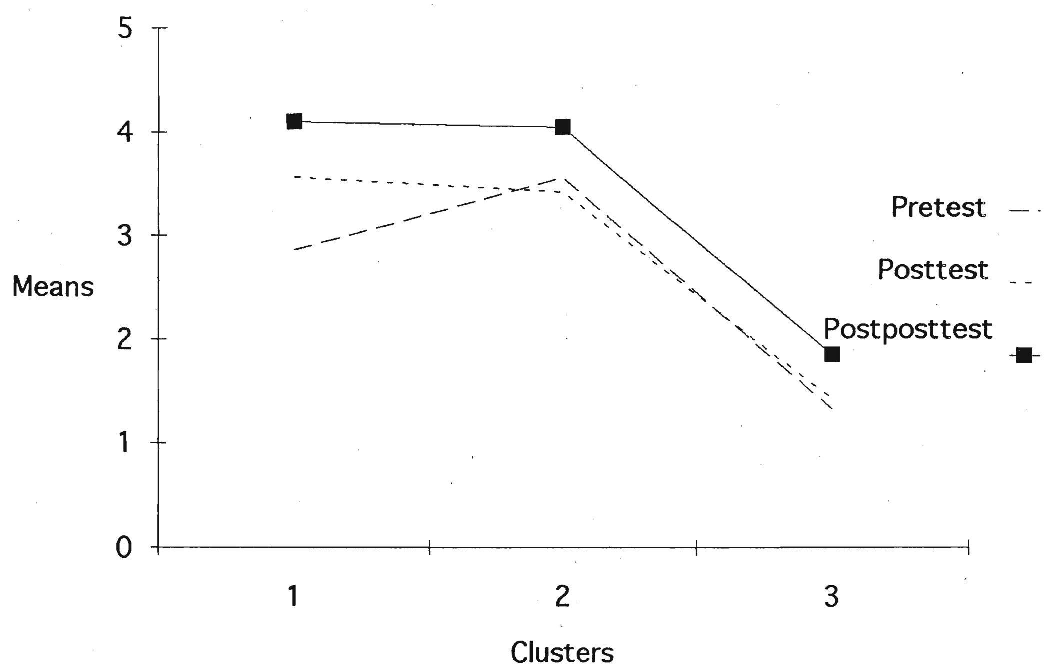
In the pretest to postposttest comparison of the control group the greatest gains were made in the exploration cluster. However, after the treatment the exploration and planning increased approximately the same amount. The conclusion/evaluation cluster increased more from the posttest to the postposttest in the control group (with the treatment) than from the pretest to the posttest in the treatment group (Figures 4 and 5).

Paired t-tests were calculated for the three cluster scores and total



(Cluster 1 = Exploration Cluster; Cluster 2 = Planning Cluster; Cluster 3 = Conclusion/Evaluation Cluster)

Figure 4. Means of the three cluster scores for the pretest and posttest of the treatment group.



(Cluster 1 = Exploration Cluster; Cluster 2 = Planning Cluster; Cluster 3 = Conclusion/Evaluation Cluster)

Figure 5. Means of the three cluster scores for the pretest, posttest, and postposttest of the control group.

test scores from the posttest and the postposttest for the control group (Table 3). The means for the conclusion/evaluation cluster and the total score in the postposttest were found to be significantly greater than the means in the posttest for the control group. Results were encouraging but not significant for the exploration and planning clusters. However, these analyses did not show the interaction effect of the treatment and trials (pretest, posttest, postposttest).

Student outcome differences in the two groups in problem solving performance as a result of teaching a problem solving strategy were assessed using a multivariate analysis of variance (MANOVA). The pretests and posttests were treated as dependent measures with the pretests as the first trial and the posttests as the second trial in the analysis. The total scores and clusters were treated as separate tests for the analysis.

The univariate F ratio for the main effect of "condition" was not significant indicating that the treatment contributed little effect on the resulting scores (Table 4). There is not sufficient evidence to show there is a difference between the group that received the instruction in the LNPSS and the group that did not (control). The F ratio for the within-subjects effect of "trials" was significant (Table 4). There was a significant increase in the scores over the two trials (pretest and posttest). The "condition by trials" effect was not significant (Table 4). It appears that there is little interaction between the treatment and the trial. However, the slope of the two lines in Figure 6 indicates that there is some interaction in this study. If these two lines were parallel it would indicate no interaction. It remains for further investigation to determine the type of interaction. The multivariate F ratio for the “condition by tests” was not significant (Table 4). Therefore the effect of the treatment over the clusters was not significant. The effect of “tests” was significant which was expected since the clusters are different from one another in value (Table 4). The “trials by tests” interaction was significant (Table 4). This means that some cluster scores across trials were significantly different. Therefore students were better at some clusters than others in the pretests as compared to the posttests. It was not determined from these results exactly what clusters were significantly different.

Teacher comments of student behaviour indicated a change over the treatment time period. The students in the treatment group took more time to complete the posttest as compared to the control group and were more engaged in the task. The students that were good at problem solving found the pretest difficult because they had to slow down and look at each step. It was difficult for them at first. After the model was introduced students that had difficulty with the problem solving said that the model helped with the solving of the problem. Once understood, the model

Table 4

MANOVA Table F Ratios for the Variables and Interactions

Source of Variation	df	F	P
Condition	1	1.03	0.316
Trials	1	4.89	0.033*
Condition by Trials	1	0.86	0.359
Condition by Tests	3	1.39	0.260
Tests	3	175.28	0.000*
Trials by Tests	3	5.65	0.003*

Condition = treatment (LNPSS instruction or control)
Trials = Pretest, Posttest
Tests = 3 clusters (Exploration Cluster, Planning Cluster, Conclusion/Evaluation Cluster) scores and total score
df = degrees of freedom
F = F ratio
P = probability value
* $p \leq 0.05$

seemed to help the students focus better on the problem.

Interpretation

Results from the pilot study utilizing two scorers indicated a high degree of interrater reliability (r = 0.94 for the total test) (Table 1) of correlation between the scorers for the cluster scores. This appears to provide strong evidence that scorers can be trained with a reasonable degree of consistency. Perhaps the small number of steps within each of the stages of the LNPSS allowed for greater agreement in the interpretation of the student results for the scorers. However, the training of the scorers during this stage was critical for the study and made the results more reliable.

The results of the t-test for independent groups for the pretests of the treatment and control groups were not significant; so it is appropriate to treat the treatment and control groups as equal for statistical purposes. However, the large SD in each group indicated a large variance in the problem solving ability of students in grade 9. In fact the range of problem solving ability was wide in the grade 9 students overall, but not significantly different across the two groups.

The increase in SD from the pretest to posttest was not expected. After the treatment there was a greater variation in the scores. Perhaps

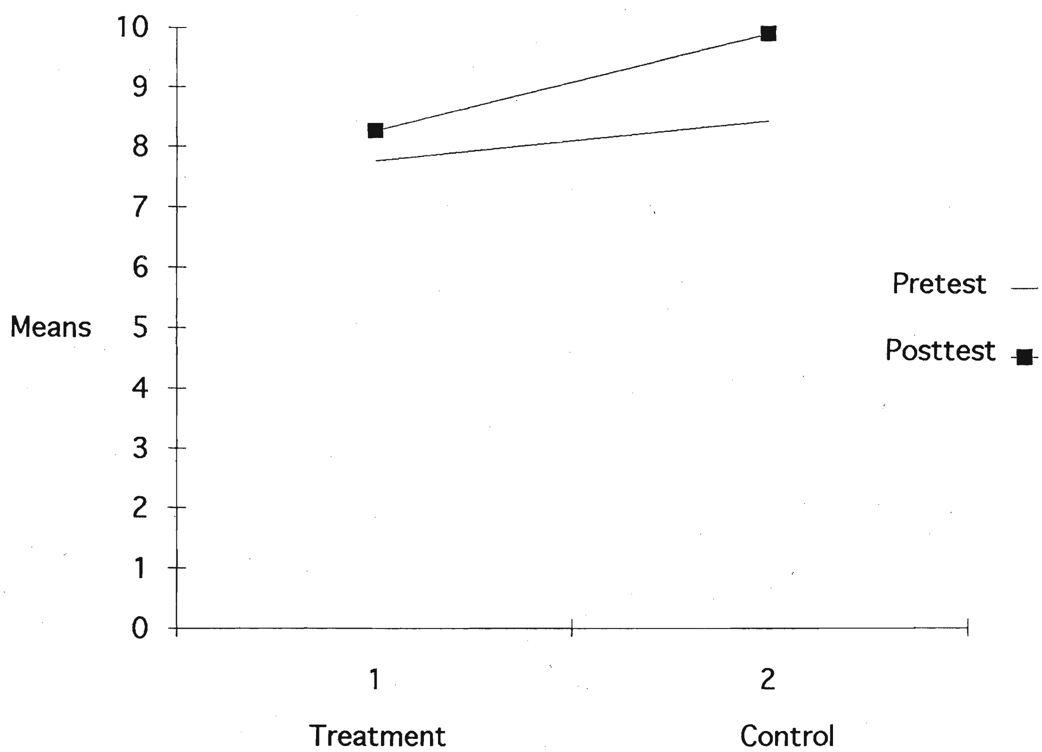


Figure 6. Means of the total scores for the pretest and posttest of the treatment and control groups.

there is another factor that is involved in the improvement of problem solving skills in students. The way in which the material was taught may not have been readily learned by some students in the class. Instead, it may have confused them enough to result in impairment of their performance. Variables such as the learning style of the students should be taken into consideration in future studies in this area. When learning something complex the learning curve demonstrates an initial drop in the achievement level and then an increase. This could result in a greater SD in the group in the early stages. It could also explain the low achievement in the posttest scores.

The decrease in SD in every cluster and total scores from the posttest to the postposttest in the control group may be an indication that the material was presented differently or that the students had similar learning styles, or perhaps there was an unidentified cognitive factor at work. The significant results in the paired t-tests for the posttests and postposttests (control group) were encouraging.

There was a noticeable trend in the data since all of the net changes in each group, that had received the treatment, were positive from the pretest to the posttest (treatment) and from the posttest to the postposttest (control). MANOVA results for the main effect of "condition" were not significant. There was a greater increase in the achievement of the problem solving tests in favour of the treatment group in all clusters but it was not significant as measured by the MANOVA. Significant results were found in the within-subjects test for "trials", which means that the posttests were significantly greater than the pretest scores, for the total group (treatment plus the control group) of students. It did show there was some interaction between the type of treatment used and the trial even though it was not significant. Since the treatment group had daily work with the LNPSS perhaps some of the increase seen in the means was because the students learned the type of test. There might also have been an unidentified cognitive factor, which was not examined, that could have had some effect on the difference in performance. This could explain the difference in increases when the treatment was applied to the control group. It was observed that the planning cluster increased about as much as the exploration cluster for the control group during the treatment. This was different from the treatment group. Perhaps the instruction was more effective the second time through by the instructor. This needs further investigation to determine whether there is something to do with the condition (treatment) itself or some other related factor.

Perhaps this was not enough time to show a difference in the level of achievement of the two groups. A study by Charles et al. (1987), that lasted 23 weeks, found that students who were instructed utilizing a problem solving approach were significantly higher in the posttest scores. It is possible that the small differences between the two groups that were beginning to become evident might have increased over time. The significant t-test results, in two areas comparing the posttest and postposttest (planning cluster and total test score for the control group) demonstrates a general trend for the results. It would be beneficial to design a longer study that would span at least a 20 week period.

The increase in the scores of both groups (control and treatment) shows that the students made the greatest percentage gains in the exploration clusters. The planning cluster also increased, but to a lesser extent. It appears that the area of exploration of problems is more readily learned by students. Also, the significant results in the paired t-tests comparing the posttest and postposttest for the control group indicate that the conclusion/evaluation cluster could be another area that can be readily learned or perhaps this cluster was taught the same way in each program.

The planning cluster seems to be a more complex stage in problem solving which requires more than simply learning a number of strategies and then applying one of them to a particular problem. During the study, the learning of the heuristics was mastered by most students. It was the application of these to new problems (planning cluster) that proved to be very difficult to learn. The coordination of the many bits of information needed to answer the problem appears to be a very complicated process to learn and may account for the lower increase in the planning cluster of the problem solving. These findings were consistent with Charles et al. (1987) which found a much slower rate of improvement in arriving at the right answer than in understanding the problem.

The observations reported by teachers is an area which needs to be expanded to reflect more about the learning that the students have experienced. The comments made indicate that the use of the model (LNPSS) was beneficial for the students that responded. This area needs to be further explored to determine the perceptions of students about their increase in problem solving achievement and how this can be effectively utilized in the evaluation of mathematics problem solving.

Summary

Results from calculation of the Pearson Product-Moment Correlation Coefficients suggest that growth strands can be reliably used in determining growth in problem solving ability. It was also clear from the data that there is a wide variance of problem solving ability in the grade 9 students but that they were not significantly different between groups.

MANOVA results were not significant for the main effect "condition". However, the significant results for the "trials by tests" effect indicate that there is some significance between the pretest and posttest scores in some of the cluster scores. Results from t-tests comparing the posttests and postposttests (control) had significant results in the conclusion/evaluation cluster and total score. This was discussed in detail and different arguments were provided. Perhaps the 6-week time period was not sufficient to show an effect by the treatment or perhaps there is something more complex working within the planning cluster that requires more time for the students to be able to apply the new knowledge to new problems. It is also possible that the cognitive ability of the students was another factor that influenced the results of the study. The inclusion of student behaviour and comments was used to a limited extent in this study with positive results. The results indicate that teaching and evaluating mathematics problem solving need to be further investigated to assist educators and researchers in understanding mathematics problem solving.

CHAPTER FIVE: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

This study focussed on the teaching and evaluation of mathematics problem solving. Initially the author developed a number of growth strands that followed the LNPSS to use in the evaluation of the level of mathematics problem solving in grade 9 students. Two classes of grade 9 students were selected as subjects for the study. A pretest utilizing the LNPSS was administered to both groups. Then a 6-week treatment began in which the one group (treatment) received instruction in the LNPSS and the use of strategies within this model for one 37-minute period each day. The control group received instruction in the regular grade 9 program using problem solving with no instruction in the use of the LNPSS. After the 6 weeks both groups were given a posttest and the control group then began the 6-week treatment. After the second 6-week instruction period the control group was given a postposttest.

Following the initial 6-week treatment a pilot study was conducted to evaluate the use of growth strands in the evaluation of mathematics problem solving. Four students were given the test by the author and evaluated by two scorers using the growth strands developed by the author. The Pearson Product-Moment Correlation Coefficients were high for all cluster scores. Following this the results from all tests were scored by two scorers utilizing the growth strands that were prepared before the study. The data were presented in tables and graphs and were analyzed using t-tests and a MANOVA table. The results from the t-tests and MANOVA table were encouraging and some results were found to be significant as described in the findings.

Conclusions

Three questions formed the basis for this study:

1. What is the range of problem solving ability levels for students in the intermediate division?

2.What are the effects of being taught a problem solving strategy upon students' abilities to solve problems?

3. Can growth in problem solving be evaluated at each stage of the problem solving process utilizing growth strands for the LNPSS?

Question 1

The author expected that the range of problem solving ability would be quite varied initially. The results demonstrated that there was a wide range in the problem solving ability level of students in grade 9 in the study. This was evident with the large SD observed for each pretest. However, it was found that the means in the two groups were not significantly different from each other in their problem solving achievement level prior to the initiation of the study.

It was expected that the planning cluster would be the most difficult for the students. However, students had difficulty with all aspects of the problem solving process. This may be only indicative of the population of students who were studied but research findings from the Ontario Ministry of Education (1990b, 1990c) showed problem solving was not well done at the intermediate level.

It was expected that after the treatment group received the treatment the SD would decrease. This was not observed and in fact the SD increased in the planning cluster and decreased in the exploration and conclusion/evaluation clusters. After the control group received the treatment the SD decreased for every cluster including the total score. The range in the problem solving ability level of the grade 9 students in the postposttest evaluation had decreased, which was expected.

<u>Question 2</u>

The students who received the instruction in the use of a problem solving model (LNPSS) increased by a greater percentage than the control group in the posttest. Differences observed between the control and the treatment groups were in the anticipated direction. However, results from the MANOVA did not demonstrate sufficient evidence to reject the null hypothesis that there is no difference between the achievement levels for students who received the instruction with the LNPSS and those who did not. However, some effects and interactions were found to be significant using the MANOVA.

In retrospect it is evident that a postposttest for the treatment group would have allowed for more comparisons against the postposttest of the control group in a MANOVA table.

Question 3

Growth strands were used successfully in this study to evaluate the growth of problem solving performance in each stage of the process. This was evident in the high correlation scores for the two scorers. Therefore it was possible to assess growth in problem solving performance using a series of observable steps for each stage of the LNPSS (growth strands) in this small sample. Use of these growth strands in further studies would provide educators with more knowledge which would allow these results to be generalized.

Implications

Implications for Practice

Even though there were not significant results for the main effect of "condition" in this study there were implications for the educator in the classroom. There is some evidence that the teaching of a problem solving strategy does increase the achievement in problem solving more than if there is no instruction in the problem solving strategy, since two areas improved significantly according to the t-test results comparing the posttest and postposttest scores (Table 3). However, this needs to be studied further with different populations and over a longer period of time to determine whether other significant results can be obtained.

Implications for Theory

Since this study has only a small sample size and is limited in the population in which it was administered the implications are not very extensive. However, it seems that there is an indication that there is something in this study that warrants further research in the area of teaching and evaluating mathematics problem solving. The interaction of the teaching of the LNPSS over time needs to be further investigated to determine the type of interaction.

Implications for Further Research

There needs to be much more study in the area of teaching and evaluating mathematics problem solving. Results from this research contribute some useful information towards the development of effective methods of teaching and evaluating mathematics problem solving. The research demonstrates that the use of a model such as the LNPSS does improve some aspects of the problem solving achievement in grade 9 students more than if they are not taught a model.

It also raises questions that need to be investigated in further studies: Can the steps for problem solving be broken down into smaller steps to assist students in increasing achievement in the planning cluster? Is a longer time required to learn these complex strategies for the planning cluster or can a different method provide a more efficient way for students to learn these processes? Since this method is so time consuming can a more practical method be utilized to use growth strands in the regular classroom? A longitudinal study involving problem solving over several grades may provide for a more gentle approach to teaching problem solving and valuable results. Further to this a comparison study between slow learners and able learners may provide useful information on the length of time to master problem solving and the processes used.

A study that utilizes growth strands to evaluate growth in problem solving skills applied over a larger sample would result in more trustworthy generalizations. This use of growth strands could prove to be a very productive area of research for evaluating growth in problem solving achievement. A study similar to this study needs to be completed over a longer time period to investigate the growth in problem solving achievement.

It is through these many small steps that we will reach this goal of building a body of knowledge for effective methods of teaching and evaluating mathematics problem solving. With the current focus on problem solving in mathematics it is a perfect time to continue research in this direction to ensure that the research will become practical.

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Appendix A

Generic Instructional Methodology

1. **The Learning**

(a) Identify the learning

(b) Identify the learning by name

2. **Plan**

(a) Identify the present level of ability (**A**)

(b) Identify the intended level (**B**)

(a) Identify the nature of the **A** to **B** transition

3. **Interact**

(a) Activate **A** -cue, review, strengthen, obliterate

(b) Activate the process for the **A** to **B** transition

(c) Identify and reinforce **B** when it occurs

(d) Consolidate **B** (embed it in a larger intellectual structure)

4. **Apply**

(a) Apply **B** to new contexts

5. **Evaluate**

(a) Assess performance with **B** in relevant contexts

(b) Assess instructional performance (adequacy of procedure for **A** to **B** i.e., transition) (Popp, 1993)

Appendix B

Pretest, Posttest, and Posttest

Class Problem Solving Pretest

Code:_____________

Maria has agreed to prepare a lasagna casserole for a party which is to be held in a neighbour's home. There will be 13 men, 11 women, 6 children, and 2 infants at the party. Maria has a recipe which serves 4 people and requires the following ingredients:

Lasagna Casserole

30 mL Salad Oil
75 mL Minced Onion
500 g Ground Chuck
1 clove Garlic
3 mL Dried Oregano
7 mL Salt
3 mL Pepper

150 mL Snipped Parsley
560 g Canned Tomatoes
225 g Canned Tomato Sauce
4 Pieces Lasagna Noodles
125 g Swiss Cheese
350 mL Cottage Cheese

As Maria climbed out of her swimming pool, she notices that it is 15:30 and the party is scheduled to start at 18:00.

(Make a list of the important facts from the information in the question.)

(Make a list of questions which occur to you as you read the problem.)

(Turn the page after you completed these tasks.)

Here is what Maria thought was important in this problem:
Maria has been asked to serve a group of people lasagna.
Maria knows there will be 13 men, 11 women, and 6 children.
A number of ingredients with varying amounts are listed for the lasagna recipe.
The time mentioned is not important for the problem.
How many recipes will Maria make?
How much of each ingredient should Maria use?
Will Maria have enough of each ingredient at her house?

(What do you think is important here?)
(Which question do you feel is central to this problem?)
(Turn the page after you have answered the questions.)

Here is the question that Maria thought was central and decided to ask:
How much of each ingredient should Maria use?

(What do you think you need to do next, before answering the question?)
(Can you show me how you will do that?)
(Turn the page after you have answered the questions.)

Here is what Maria thought was important information that she needed to solve the problem:

Maria will have to know how many people to serve.
There are 13 men, 11 women, and 6 children. The 2 infants will not eat lasagna.
1 recipe serves 4 people
2 recipes serves __ people
______ recipes serves ____.

(How will you determine the number of people to be served?)
(How will you determine the relationships between the number of recipes and the number of people to be served?)
(Turn the page when you have filled in the blanks.)

Here is how Maria answered the questions:

She will have to serve 30 people. There are 13 men, 11 women, and 6 children who will be eating the lasagna. $13 + 11 + 6 = 30$

$30 \div 4 = 7.5$, so Maria needs 7.5 recipes.

(How will you demonstrate how to find the rest of the information which you need to answer the question?)
(Turn the page when you have answered the question.)

Here is how Maria chose to represent her information:

Ingredients	Salad Oil	Minced Onion	Ground Beef
1 recipe	30 mL	75 mL	500 g
7.5 recipes	?	?	?

(How would you find the answers to the blanks?)
(Calculate your results.)
(Turn the page after you have answered the questions.)

Here are some of the calculations Maria completed for her problem:

30	75	500
x7.5	x7.5	x7.5
225	562.5	3750

(What did Maria find out?)

(Turn the page after you have answered the question.)

Here is what Maria found out:

225 mL Salad Oil
562.5 mL Minced Onion
3750 g Ground Chuck
7.5 clove Garlic
22.5 mL Dried Oregano
52.5 mL Salt
22.5 mL Pepper
1125 mL Snipped Parsley
4200 g Canned Tomatoes
1687.5 g Canned Tomato Sauce
30 Pieces Lasagna Noodles
937.5 g Swiss Cheese
2625 mL Cottage Cheese

(Does this answer the question and solve the problem which we started out with in the beginning?)
(How can you be sure?)
(Are the amounts for each ingredient reasonable? How could you be sure that they are?)
(Can you think of another way to represent the information instead of the table?)
(Could you think of another method to solve this problem more efficiently?)

Class Problem Solving Posttest

Code:_____________

Class Problem Solving Posttest #1 and Postposttest #2

The Grad Committee at McKinnon Park Secondary School has determined that they need $100.00 to keep the Grad Dance from going in the red. After some checking, they found that $10.40 was left over from last year's dance. The committee members discussed the situation and finally decided to make up "Fresh Fruit from Florida" baskets which they figure they can easily sell to raise the necessary amount.
After making a number of inquires, the committee's treasurer figures that they can make $2.50 on each full case of oranges they use and $3.60 on each full case of grapefruit. They need a lot more oranges than grapefruit in making up the baskets-- in fact, they use two full cases of oranges for every case of grapefruit.
Tommy Frederickesn's father is arranging for the cases to be delivered to the school for a flat fee of $5.00.

(Make a list of the important facts from the information in the question.)

(Make a list of questions which occur to you as you read the problem.)

(Turn the page after you have completed these tasks.)

Here is what the Grad Committee thought was important in this problem:
The Grad Committee needs $100.00 and they have decided to sell fruit baskets to raise this money.
They know how much money they have from last year.
They know how much they will make on each case and the delivery charges.
How many cases of oranges will they need?
How many cases of grapefruit will they need?
How many cases of grapefruit and oranges will they need?
When they reach their goal will there be any partially used cases left over (wasted)?
Is $5.00 a reasonable delivery charge?

(What do you think is important here?)
(Which question do you feel is central to this problem?)
(Turn the page after you have answered these questions.)

Here is the question that the Grad Committee thought was central and decided to ask:

How many cases of grapefruit and oranges will they need?

(What do you think you need to do next, before answering the question?)

(Can you show me how you will do that?)

(Turn the page after you have answered the questions.)

Here is what the Grad Committee thought was important information that they needed to solve the problem:

They need $100.00.
The money they have coming in is: money left over from last year and the money from the sale of the fruit.
The only money they have going out is the delivery charge.
When they take their expenses away from their income they want to end up with $100.00.

(What other information do they need to know to answer the question?)
(What are some relationships between elements in the problem?)
(Turn the page when you have answered these questions.)

Here is more information relationships they know:
For every case of grapefruit they use 2 cases of oranges.
They make \$3.60 on each case of grapefruit.
They make \$2.50 on each case of oranges.
They end up with \$100.00.

(How will you demonstrate how to find the rest of the information which you need to answer the question?)
(Turn the page when you have answered these question.)

Here is how the Grad Committee chose to represent the information:

Income	-	Expenses	= \$100.00
(Last year's surplus) + (Profits from fruit sales)	-	(Delivery charge)	= \$100.00
(\$10.40 + profits _____)	-	\$5.00	= \$100.00

How can we determine the number of cases used.

We can let the number of cases of grapefruit used be N. Then the number of cases of oranges would be ______.

The profits will be:

\$3.60 x N +(\$2.50 x ______)

(How would you find the answers to the blanks?)

(Calculate your results.)

(Turn the page after you have answered the questions.)

Here are some of the calculations they completed for the problem:

If there are N cases of grapefruit there are 2N cases of oranges used.
Profits = \$3.60 x N + \$2.50 x 2N

\$10.40 + (profits) - \$5.00 = \$100.00
\$10.40 + (\$3.60 x N + \$2.50 x 2N) - \$5.00 = \$100.00
\$10.40 + (\$3.60N + \$5.00N) - \$5.00 = \$100.00
\$10.40 + \$8.60N - \$5.00 = \$100.00
\$5.40 + \$8.60N = \$100.00
\$8.60N =\$94.60
N = 11
(What did you find out?)
(How many cases of grapefruit and oranges were used?)
(Turn the page after you have answered the question.)

Here is what they found out:

There were 11 cases of grapefruit and 2 x 11 or 22 cases of oranges needed.

(Does this answer the question and solve the problem which we started out with in the beginning?)
(How can you be sure?)
(Are the amounts for each fruit reasonable? How could you be sure that they are?)
(Can you think of another way to represent the information instead of the table?)
(Could you think of another method to solve this problem more efficiently?)

Class Problem Solving Posttest

Code:_____________

Class Problem Solving Posttest #2 and Postposttest #1

To go from his office to his home, Mr. Robinson travels 20 km down a main highway, and then 10 km down a side road. At 4:50, when Mr. Robinson is about to leave his office, he notices that it is snowing very hard and that the wind is blowing. With this blizzard, he wonders whether he will be able to make it home down his side road. He phones his son who agrees to meet in the ski-doo where the highway meets the side road. It will take Mr. Robinson 15 minutes to get his hat and coat on and his car started, and it will take his son 15 minutes to get the ski-doo ready to go. The ski-doo travels 40 km/h and the car travels 80 km/h.

(Make a list of the important facts from the information in the question.)
(Make a list of questions which occur to you as you read the problem.)
(Turn the page after you completed these tasks.)

Here is what I thought was important in this problem:
Mr. Robinson is ready to go home.
The son has to meet Mr. Robinson.
Know distances, time of day, speed of vehicles, times to get ready to leave.
When will Mr. Robinson leave?
At what time should the son start to get ready?
When will they meet?
Will either have to wait in the snow?

(What do you think is important here?)
(Which question do you feel is central to this problem?)
(Turn the page after you have answered the questions.)

Here is the question that I thought was central and decided to ask:
At what time should the son start to get ready?

(What do you think you need to do next, before answering the question?)
(Can you show me how you will do that?)
(Turn the page after you have answered these questions.)

Here is what I thought was important information that I needed to solve the problem:

Mr. Robinson is getting ready to leave at 4:50.
He will be ready to leave in 15 minutes.
He will drive 20 km at 80 km/h.
The son will drive 10 km at 40 km/h.
It will take the son 15 minutes to get ready to leave.

(How will you determine the time it takes for Mr. Robinson to reach the corner?)
(How will you determine the relationships between the time Mr. Robinson takes to reach the corner and the time his son takes?)
(Turn the page when you have answered the questions.)

Here is how I answered the questions:

d = distance, r = rate of travel or speed, t = time

$d = r \times t$, so $t = d/r$ 60 min. = 1 h

Total time for Mr. Robinson to reach the corner = Total time for son to reach the corner. (Otherwise, one of them will have to wait in the snow!)

(How will you demonstrate how to find the rest of the information which you need to answer the question?)

(Turn the page when you have answered the question.)

Here is how I chose to represent my information:

```
                 Now(4:50)       Leaves
Mr. Robinson       _____|___________|__________20 km  @ 80 km/h_____
                  |<  15 min  >|

Son                ________|___________|_______10 km  @ 40 km/h_____
                  Starts?         Leaves?
                      |<  15 min  >|
```

A. Travel time (Mr. Robinson) : 15 min + (20/80) x 60 = ________

B. Travel time (son) : 15 min + (10/40) x 60 = ________

C. Time Mr. Robinson gets to the corner: 4:50 + A = __________

D. Time son gets ready: C - B. = __________

(How would you find the answers to the blanks?)

(Calculate your results.)

(Turn the page after you have answered the questions.)

Here are some of the calculations I completed for the problem:

15 min. + (20/80) = 15 min. + 0.25 h x 60 (change hours to min 0.25 x 60 = 15)
= 15 min. + 15 min.
= 30 min.

Time Mr. Robinson gets to the corner: 4:50 + 30 min. = 5:20

20 min. + (10/40) x 60 = 15 min. + 0.25 h x 60
(change hours to min 0.25 x 60 = 15)
= 15 min. + 15 min.
= 30 min.

Son's travel time: 30 minutes.

(What did you find out?)
(Turn the page after you have answered the question.)

Here is what I found out:

Time son gets ready is 5:20 - 30 = 4:50

The son will have to leave at 4:50 to reach the corner the same time as his dad.

(Does this answer the question and solve the problem which we started out with in the beginning?)

(How can you be sure)?)

(Are the amounts for each travel time reasonable? How could you be sure that they are?)

(Can you think of another way to represent the information instead of the diagram?)

(Could you think of another method to solve this problem more efficiently?)

Appendix C

Growth Strands For The Logical Numerical Problem Solving Strategy

(Adapted from Borthwick & Fowler, 1989; Eagan, 1993; Popp, 1986)

1. **Exploration of the Problem**

Score

3 4. Student engages in purposeful and focussed exploration with competence in arrangement and classification of information. Student identifies important information and classifies it at this stage.

2 3. Student engages in purposeful exploration which leads to a focus. The student gathers information but it is not organized in any manner.

1 2. Student engages in random exploration but never focused. He/She chooses some irrelevant information from the event.

0 1. Student engages in imitative, exploratory behaviour. Student is not able to identify important information in the event.

2. Question

Score

2 3. Student poses a question linking the two or more factors or concepts in the event.

1 2. Student poses a global question about the event, or a general question about a single factor.

0 1. Student observes the event, but does not indicate any recognition of individual characteristics. No relationships are hinted at in a question. Makes general statements.

3. **Organization**

(a) analysis

(b) information

The total score for this stage is the sum of scores for this section and the representation section.

Score

2 3. Student searches purposefully for additional sources of information. There is a list of focussed information.

1 2. Student searches randomly for additional information and provides a list of information.

0 1. Student employs immediate familiar sources of information and knowledge of observable characteristics of the event. There is no list of the information.

3. **Organization** (Continued)

(c) representation

Score

2 3. Student uses a diagrammatic picture which includes mostly relevant elements.

1 2. Student uses a picture which includes relevant elements.

0 1. Picture focussing on irrelevant elements in the problem (e.g., Kindergarten painting).

4. **Calculation**

Score

3 4. Student employs appropriate operations and formulae to solve the problem.

2 3. Student employs combinations of the four basic operations to solve the problem.

1 2. Student employs one of the four basic operations to solve the problem.

0 1. Student employs no particular operation to solve the problem.

5. Conclusion

Score

2 3. Student elaborates the statement and provides implications of the solution for other problems.

1 2. Student provides a simple statement of the correct conclusion.

0 1. No conclusion is offered for the problem.

6. Record

Score

1 2. Student records implications (e.g., about procedures used) which would be helpful in future.

0 1. Student provides a record of only the answer.

7. Evaluation

Score

2 3. Student evaluates the appropriateness of the answer on two levels: in relation to the original question and the reasonableness of the answer.

1 2. Student evaluates all calculations in the answer or relates answer to original question.

0 1. Student provides no evidence for evaluation of the answer.

Appendix D

Heuristics Taught During the Study And Embedded In The Logical Numerical Problem Solving Strategy

1. **Restate problem** in your own words.
2. Decide on **what information is relevant and irrelevant.**
3. **Determine a question** for the problem situation.
4. **Draw a diagram** of the problem situation.
5. **Make a chart or table** using the information from the problem and show the relationships of the elements to each other.
6. **Look for a pattern** in the numbers that you arrange in charts or tables.
7. **Simplify the problem** by substituting smaller numbers for the larger numbers in the problem. This is like doing an experiment using different numbers.
8. **Guess and check**. Take a guess and check to see if it is close.
9. **Work backwards**. In this one the final value may be given and the student is asked to find the beginning value.
10. **Similar problems**. Think of other problems you have worked on that are like the one you are doing in terms of conditions.
11. **Make an organized list** of the relevant information.
12. **Elimination** (deductive logic)

Appendix E

Problems

A farmer has hens and goats. When she looks in the field she sees 50 heads. When she looks through the fence she sees 140 feet (Pólya, 1962, p. 23).

Fifteen boys in a class rented the arena for $10.50 for fifteen minutes. Tom's mother offered to pay for either the bus fare which was $1.00 each way or Tom's share of the rental (Chadwick, 1984, p. 29).

Paula has collected 14 foreign coins. Mandy has collected 6 more than twice as many as Paula (Chadwick, 1984, p. 33).

A train one kilometre long travels at the rate of ten kilometres per hour. As it travels through the Rocky Mountains it enters a tunnel which is one kilometre in length (Chadwick, 1984, p. 73).

A jar has a lid which is 6 cm in diameter. The jar is 8 cm high. At the bottom of the jar there is a caterpillar. Each day the caterpillar crawls up 4 cm. Each night it falls down 2 cm (Chadwick, 1984, p. 74).

Tanya has saved $24.00. She intends to buy an outfit for $78.50. She earns $5.00 per hour part time at the grocery store (Chadwick, 1984, p. 53).

A fireman stood on the middle step of a ladder, directing water into a burning building. As the smoke cleared, he climbed up 3 steps and continued to work. The fire got worse so he had to go down 5 steps. Later, he climbed up the last 6 steps and was at the top of the ladder (Chadwick, 1984, p. 75).

Jennifer wants to fence a square garden that is 6 m wide. She wants to place the fence posts 2 m apart (Chadwick, 1984, p. 76).

An owner of a new condominium complex wanted to place a new refrigerator and stove in each of the 96 units. She spent a total of $158,400.00. All of the refrigerators cost $105,600.00 (Chadwick, 1984, p. 100).

Sally, an avid canoeist, decided one day to paddle upstream 6 km. In 1 hour (h), she could travel 2 km upstream using her strongest stroke. After such strenuous activity, she needed to rest for 1 h, during which time the canoe floated downstream 1 km. In this manner of paddling for 1 h and resting for 1 h, she travelled 6 km upstream.

There are eighteen people in a room. Each person shakes hands with each of the other people once and only once (Krulick & Rudnick, 1989, p. 129).

Three boxes each contain a number of billiard balls. One box contains only even-numbered billiard balls, one box contains only odd-numbered billiard balls, and the third contains a mixture of odd- and even-numbered balls. All of the boxes are mislabelled (Krulick & Rudnick, 1989, p. 141).

Three boys stood on a scale and put a nickel in the slot. The scale showed 150 kg as the total mass. One boy stepped off the scale. It showed 82 kg. The second boy stepped off the scale, and it showed 45 kg (Krulick & Rudnick, 1989, p. 133).

A grocer has three pails, an empty pail that holds 5 litres, an empty pail that holds 3 litres, and an 8 litre pail that is filled with apple cider. The grocer wants to give a customer exactly 4 litres of cider (Krulick & Rudnick, 1989, p. 143).

There are 16 football teams in the National Football League. To conduct their annual draft, teams in each city must have a direct phone line to each of the other cities. Suppose the league expanded to 24 teams (Krulick & Rudnick, 1989, p. 167).